Mplus Short Courses
Topic 8

Multilevel Modeling With Latent Variables Using Mplus: Longitudinal Analysis

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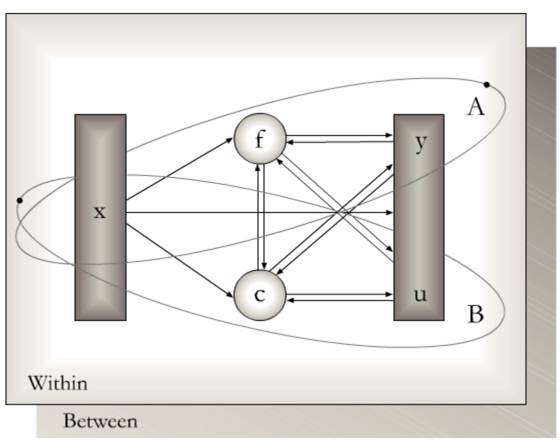
Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
  - V5: November 2007
  - V5.2: November 2008
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn, Jean Maninger

General Latent Variable Modeling Framework



Mplus

Several programs in one

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis
- Survival analysis
- Growth modeling
- Multilevel analysis
- Complex survey data analysis
- Monte Carlo simulation

Fully integrated in the general latent variable framework

Overview Of Mplus Courses

- **Topic 1.** March 18, 2008, Johns Hopkins University: Introductory - advanced factor analysis and structural equation modeling with continuous outcomes
- **Topic 2.** March 19, 2008, Johns Hopkins University: Introductory - advanced regression analysis, IRT, factor analysis and structural equation modeling with categorical, censored, and count outcomes
- **Topic 3.** August 21, 2008, Johns Hopkins University: Introductory and intermediate growth modeling
- **Topic 4.** August 22, 2008, Johns Hopkins University: Advanced growth modeling, survival analysis, and missing data analysis

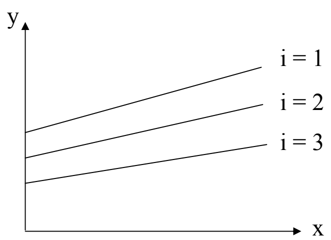
Overview Of Mplus Courses (Continued)

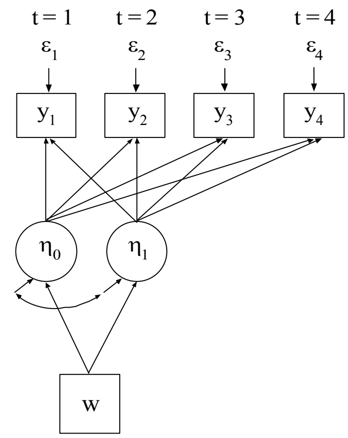
- **Topic 5.** November 10, 2008, University of Michigan, Ann Arbor: Categorical latent variable modeling with cross-sectional data
- **Topic 6.** November 11, 2008, University of Michigan, Ann Arbor: Categorical latent variable modeling with longitudinal data

- **Topic 7.** March 17, 2009, Johns Hopkins University: Multilevel modeling of cross-sectional data
- **Topic 8.** March 18, 2009, Johns Hopkins University: Multilevel modeling of longitudinal data

Multilevel Growth Models

Individual Development Over Time





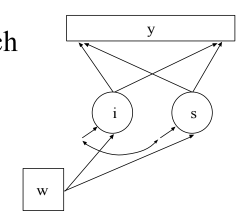
(1) $y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}$

(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- Wide: Multivariate, Single-Level Approach

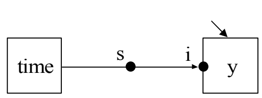


$y_{ti} = i_i + s_i \times \text{time}_{ti} + \varepsilon_{ti}$

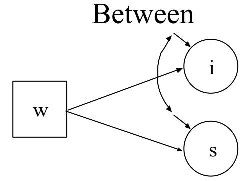
i_i regressed on w_i
s_i regressed on w_i

- Long: Univariate, 2-Level Approach (CLUSTER = id)

Within



Between



The intercept i is called y in Mplus

Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long (Continued)

- Wide (one person):

		t1	t2	t3	t1	t2	t3	
Person i:	id	y1	y2	y3	x1	x2	x3	w

- Long (one cluster):

Person i:	t1	id	y1	x1	w
	t2	id	y2	x2	w
	t3	id	y3	x3	w

Mplus command: DATA LONGTOWIDE (UG ex 9.16)
DATA WIDETOLONG

Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

Growth Models With Categorical Outcomes

Growth Model With Categorical Outcomes

- Individual differences in development of probabilities over time
- Logistic model considers growth in terms of log odds (logits), e.g.

(1) $$\log\left[\frac{P(u_{ti}=1 \mid \eta_{0i}, \eta_{1i}, \eta_{2i}, x_{ti})}{P(u_{ti}=0 \mid \eta_{0i}, \eta_{1i}, \eta_{2i}, x_{ti})}\right] = \eta_{0i} + \eta_{1i} \cdot (x_{ti} - c) + \eta_{2i} \cdot (x_{ti} - c)^2$$

for a binary outcome using a quadratic model with centering at time c. The growth factors η_{0i}, η_{1i}, and η_{2i} are assumed multivariate normal given covariates,

(2a) $\eta_{0i} = \alpha_0 + \gamma_0 \, w_i + \zeta_{0i}$

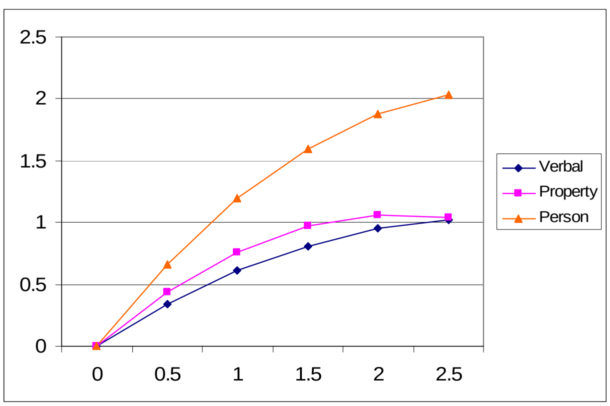
(2b) $\eta_{1i} = \alpha_1 + \gamma_1 \, w_i + \zeta_{1i}$

(2c) $\eta_{2i} = \alpha_2 + \gamma_2 \, w_i + \zeta_{2i}$

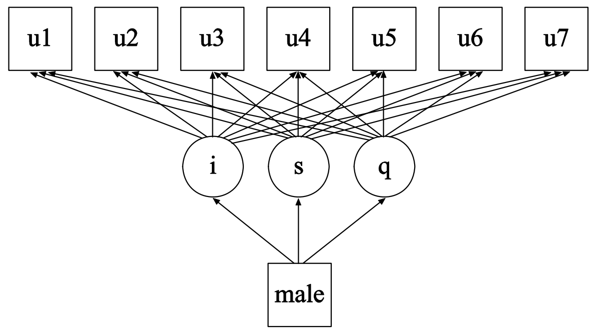
Aggression Growth Analysis

- Baltimore cohort 1: 1174 students in 41 classrooms (clustering due to classroom initially ignored)
- 8 time points over grades 1-7
- Quadratic growth
- Dichotomized items from the aggression instrument
- 4 analyses
  - Single item ("Breaks Things"), ignoring clustering (ML uses 3 dimensions of numerical integration)
  - Multiple indicators (Property aggression factor), ignoring clustering (WLSM)
  - Single item with clustering (WLSM)
  - Multiple indicators with clustering (WLSM)

Different Growth Curves For Different Aspects Of Aggression



Binary Growth Modeling Of Aggression Item (No Clustering)



Input Binary Growth Modeling

```
TITLE:     Hopkins Cohort 1 All time points with Classroom
           Information
DATA:      FILE = Cohort1_classroom_ALL.DAT;
VARIABLE:  NAMES ARE PRCID

           stub1F bkRule1F harmO1F bkThin1F yell1F takeP1F fight1F
           lies1F tease1F

           stub1S bkRule1S harmO1S bkThin1S yell1S takeP1S fight1S
           lies1S tease1S

           stub2S bkRule2S harmO2S bkThin2S yell2S takeP2S fight2S
           lies2S tease2S

           stub3S bkRule3S harmO3S bkThin3S yell3S takeP3S fight3S
           lies3S tease3S
```

Input Binary Growth Modeling (Continued)

```
stub4S bkRule4S harmO4S bkThin4S yell4S takeP4S
fight4S lies4S tease4S

stub5S bkRule5S harmO5S bkThin5S yell5S takeP5S
fight5S lies5S tease5S

stub6S bkRule6S harmO6S bkThin6S yell6S takeP6S
fight6S lies6S tease6S

stub7S bkRule7S harmO7S bkThin7S yell7S takeP7S
fight7S lies7S tease7S

gender race des011 sch011 sec011 juv99 violchld
antisocr conductr

athort1F harmP1S athort1S harmP2S athort2S harmP3S
athort3S harmP4S athort4S harmP5S athort5S harmP6S
harmP7S athort7S
```


Input Binary Growth Modeling (Continued)

```
            stub2F bkRule2F harmO2F bkThin2F yell2F takeP2F
            fight2F harmP2F lies2F athort2F tease2F
            classrm;
!           CLUSTER = classrm;
            USEVAR = bkthin1f bkthin1s bkthin2s bkthin3s bkthin4s
            bkthin5s bkthin6s bkthin7s male;
            CATEGORICAL = bkthin1f - bkthin7s;
            MISSING = ALL (999);
DEFINE:     CUT bkThin1f(1.5);
            CUT bkThin1s(1.5);
            CUT bkThin2s(1.5);
            CUT bkThin3s(1.5);
            CUT bkThin4s(1.5);
            CUT bkThin5s(1.5);
            CUT bkThin6s(1.5);
            CUT bkThin7s(1.5);
            male = 2 - gender;
```

Input Binary Growth Modeling (Continued)

```
ANALYSIS:    INTEGRATION = 10;
             PROCESS = 4;
             ESTIMATOR = MLR;
MODEL:       i s q | bkthin1f@0 bkthin1s@.5 bkthin2s@1.5
             bkthin3s@2.5 bkthin4s@3.5 bkthin5s@4.5 bkthin6s@5.5
             bkthin7s@6.5;
             i-q ON male;
OUTPUT:      TECH1 TECH8 STANDARDIZED;
```


Output Excerpts Binary Growth (No Clustering)

```
Tests of Model Fit

Loglikelihood
      H0 Value                                      -3546.377
      H0 Scaling Correction Factor for MLR             1.0005
Information Criteria
      Number of Free Parameters                            12
      Akaike (AIC)                                   7116.754
      Bayesian (BIC)                                 7177.572
      Sample-Size Adjusted BIC                       7139.455
         (n* = (n + 2) / 24)
```

Output Excerpts Binary Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
i |				
bkthin1f	1.000	0.000	999.000	999.000
bkthin1s	1.000	0.000	999.000	999.000
bkthin2s	1.000	0.000	999.000	999.000
bkthin3s	1.000	0.000	999.000	999.000
bkthin4s	1.000	0.000	999.000	999.000
bkthin5s	1.000	0.000	999.000	999.000
bkthin6s	1.000	0.000	999.000	999.000
bkthin7s	1.000	0.000	999.000	999.000

Output Excerpts Binary Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
s |				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.500	0.000	999.000	999.000
bkthin2s	1.500	0.000	999.000	999.000
bkthin3s	2.500	0.000	999.000	999.000
bkthin4s	3.500	0.000	999.000	999.000
bkthin5s	4.500	0.000	999.000	999.000
bkthin6s	5.500	0.000	999.000	999.000
bkthin7s	6.500	0.000	999.000	999.000

Output Excerpts Binary Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
q |				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.250	0.000	999.000	999.000
bkthin2s	2.250	0.000	999.000	999.000
bkthin3s	6.250	0.000	999.000	999.000
bkthin4s	12.250	0.000	999.000	999.000
bkthin5s	20.250	0.000	999.000	999.000
bkthin6s	30.250	0.000	999.000	999.000
bkthin7s	42.250	0.000	999.000	999.000

Output Excerpts Binary Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
i ON				
male	1.109	0.186	5.960	0.000
s ON				
male	-0.137	0.121	-1.132	0.258
q ON				
male	0.028	0.019	1.494	0.135
s WITH				
i	-1.298	0.314	-4.137	0.000
q WITH				
i	0.138	0.040	3.435	0.001
s	-0.059	0.026	-2.260	0.024

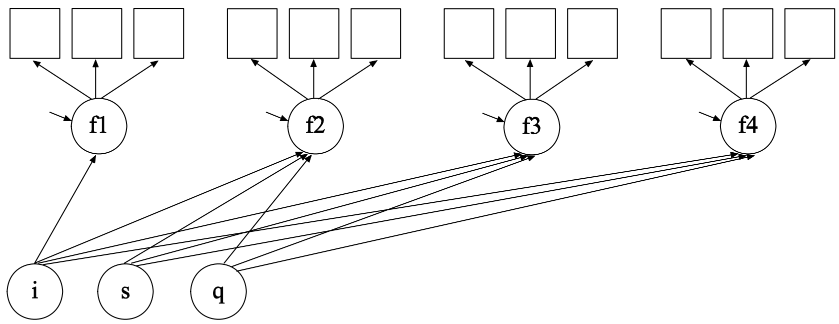
Output Excerpts Binary Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Intercepts				
I	0.000	0.000	999.000	999.000
S	0.204	0.102	1.998	0.046
Q	-0.045	0.017	-2.689	0.007
Thresholds				
bkthin1f$1	1.839	0.149	12.324	0.000
bkthin1s$1	1.839	0.149	12.324	0.000
bkthin2s$1	1.839	0.149	12.324	0.000
bkthin3s$1	1.839	0.149	12.324	0.000
bkthin4s$1	1.839	0.149	12.324	0.000

Output Excerpts Binary Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
bkthin5s$1	1.839	0.149	12.324	0.000
bkthin6s$1	1.839	0.149	12.324	0.000
bkthin7s$1	1.839	0.149	12.324	0.000
Residual Variances				
i	3.803	0.625	6.083	0.000
s	0.545	0.190	2.868	0.004
q	0.007	0.004	1.781	0.075

Multiple Indicator Growth (No Clustering)



Input Excerpts Multiple Indicator Growth (No Clustering)

```
          USEVAR = bkThin1f bkThin1s bkThin2s bkThin3s bkThin4s
          bkThin5s bkThin6s bkThin7s harm01f harm01s harm02s
          harm03s harm04s harm05s harm06s harm07s takeP1f takeP1s
          takeP2s takeP3s takeP4s takeP5s takeP6s takeP7s male;
          CATEGORICAL = bkThin1f - takeP7s;
          MISSING = ALL (999);
DEFINE:   CUT bkThin1f(1.5);
          CUT bkThin1s(1.5);
          CUT bkThin2s(1.5);
          CUT bkThin3s(1.5);
          CUT bkThin4s(1.5);
          CUT bkThin5s(1.5);
          CUT bkThin6s(1.5);
          CUT bkThin7s(1.5);

          CUT harm01f (1.5);
          CUT harm01s (1.5);
          CUT harm02s (1.5);
          CUT harm03s (1.5);
```

Input Excerpts Multiple Indicator Growth (No Clustering) (Continued)

```
CUT harmO4s (1.5);
CUT harmO5s (1.5);
CUT harmO6s (1.5);
CUT harmO7s (1.5);

CUT takeP1f(1.5);
CUT takeP1s(1.5);
CUT takeP2s(1.5);
CUT takeP3s(1.5);
CUT takeP4s(1.5);
CUT takeP5s(1.5);
CUT takeP6s(1.5);
CUT takeP7s(1.5);

male = 2 - gender;
```


Input Excerpts Multiple Indicator Growth (No Clustering)

```
ANALYSIS:     PROCESS = 4;
              ESTIMATOR = WLSM;
              PARAMETERIZATION = THETA;
MODEL:        f1 BY bkthin1f
              harmo1f (1)
              takep1f (2);
              f2 BY bkthin1s
              harmo1s (1)
              takep1s (2);
              f3 BY bkthin2s
              harmo2s (1)
              takep2s (2);
```

Input Excerpts Multiple Indicator Growth (No Clustering) (Continued)

```
f4 BY bkthin3s
harmo3s (1)
takep3s (2);
f5 BY bkthin4s
harmo4s (1)
takep4s (2);
f6 BY bkthin5s
harmo5s (1)
takep5s (2);
f7 BY bkthin6s
harmo6s (1)
takep6s (2);
```


Input Excerpts Multiple Indicator Growth (No Clustering) (Continued)

```
             f8 BY bkthin7s
             harmo7s (1)
             takep7s (2);

             [bkthin1f$1-bkthin7s$1] (11);
             [harmo1f$1-harmo7s$1] (12);
             [takep1f$1-takep7s$1] (13);

             i s q | f1@0 f2@.5 f3@1.5 f4@2.5 f5@3.5 f6@4.5
             f7@5.5 f8@6.5;
             i-q ON male;

OUTPUT:      TECH1 TECH8 STANDARDIZED;
```

Output Excerpts Multiple Indicator Growth (No Clustering)

```
Test of Model Fit

Chi-Square Test of Model Fit
          Value                              675.348*
          Degrees of Freedom                      300
          P-Value                              0.0000
          Scaling Correction Factor for WLSM    0.797

*         The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and
          WLSMV cannot be used for chi-square difference tests.
          MLM, MLR and WLSM chi-square difference testing is
          described in the Mplus Technical Appendices at
          www.statmodel.com. See chi-square difference testing in
          the index of the Mplus Users' Guide.
```


Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

```
Chi-Square Test of Model Fit for the Baseline Model
          Value                             37306.671
          Degrees of Freedom                      300
          P-Value                              0.0000

CFI/TLI
          CFI                                   0.990
          TLI                                   0.990

Number of Free Parameters                          24
RMSEA (Root Mean Square Error of Approximation)
          Estimate                              0.033
```

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f1 BY				
bkthin1f	1.000	0.000	999.000	999.000
harmo1f	1.029	0.013	77.184	0.000
takep1f	1.009	0.013	80.131	0.000
f2 BY				
bkthin1s	1.000	0.000	999.000	999.000
harmo1s	1.029	0.013	77.184	0.000
takep1s	1.009	0.013	80.131	0.000
f3 BY				
bkthin2s	1.000	0.000	999.000	999.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
harmo2s	1.029	0.013	77.184	0.000
takep2s	1.009	0.013	80.131	0.000
f4 BY				
bkthin3s	1.000	0.000	999.000	999.000
harmo3s	1.029	0.013	77.184	0.000
takep3s	1.009	0.013	80.131	0.000
f5 BY				
bkthin4s	1.000	0.000	999.000	999.000
harmo4s	1.029	0.013	77.184	0.000
takep4s	1.009	0.013	80.131	0.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f6 BY					
	bkthin5s	1.000	0.000	999.000	999.000
	harmo5s	1.029	0.013	77.184	0.000
	takep5s	1.009	0.013	80.131	0.000
f7 BY					
	bkthin6s	1.000	0.000	999.000	999.000
	harmo6s	1.029	0.013	77.184	0.000
	takep6s	1.009	0.013	80.131	0.000
f8 BY					
	bkthin7s	1.000	0.000	999.000	999.000
	harmo7s	1.029	0.013	77.184	0.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
	takep7s	1.009	0.013	80.131	0.000
i ON					
	male	0.445	0.064	6.963	0.000
s ON					
	male	0.018	0.044	0.405	0.685
q ON					
	male	0.000	0.007	0.020	0.984
Intercepts					
	f1	0.000	0.000	999.000	999.000
	f2	0.000	0.000	999.000	999.000
	f3	0.000	0.000	999.000	999.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f4	0.000	0.000	999.000	999.000
f5	0.000	0.000	999.000	999.000
f6	0.000	0.000	999.000	999.000
f7	0.000	0.000	999.000	999.000
f8	0.000	0.000	999.000	999.000
i	0.000	0.000	999.000	999.000
s	-0.025	0.033	-0.762	0.446
q	-0.002	0.005	-0.353	0.724
Thresholds				
bkthin1f$1	0.775	0.049	15.867	0.000
bkthin1s$1	0.775	0.049	15.867	0.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
bkthin2s$1	0.775	0.049	15.867	0.000
bkthin3s$1	0.775	0.049	15.867	0.000
bkthin4s$1	0.775	0.049	15.867	0.000
bkthin5s$1	0.775	0.049	15.867	0.000
bkthin6f$1	0.775	0.049	15.867	0.000
bkthin7s$1	0.775	0.049	15.867	0.000
harmo1f$1	0.488	0.049	9.961	0.000
harmo1s$1	0.488	0.049	9.961	0.000
harmo2s$1	0.488	0.049	9.961	0.000
harmo3s$1	0.488	0.049	9.961	0.000
harmo4s$1	0.488	0.049	9.961	0.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
harmo5f$1	0.488	0.049	9.961	0.000
harmo6s$1	0.488	0.049	9.961	0.000
harmo7s$1	0.488	0.049	9.961	0.000
takep1f$1	0.489	0.049	10.153	0.000
takep1s$1	0.489	0.049	10.153	0.000
takep2s$1	0.489	0.049	10.153	0.000
takep3s$1	0.489	0.049	10.153	0.000
takep4s$1	0.489	0.049	10.153	0.000
takep5s$1	0.489	0.049	10.153	0.000
takep6s$1	0.489	0.049	10.153	0.000
takep7s$1	0.489	0.049	10.153	0.000

Output Excerpts Multiple Indicator Growth (No Clustering) (Continued)

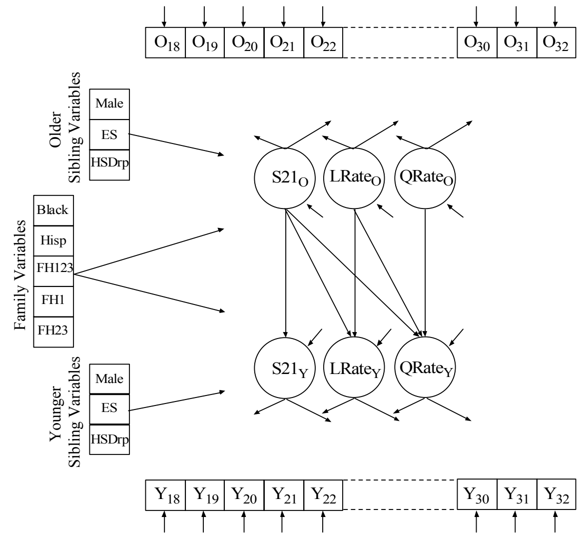
Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Residual Variances				
f1	0.164	0.031	5.268	0.000
f2	0.268	0.023	11.505	0.000
f3	0.506	0.027	18.452	0.000
f4	0.511	0.033	15.715	0.000
f5	0.452	0.034	13.203	0.000
f6	0.468	0.031	14.989	0.000
f7	0.424	0.033	12.731	0.000
f8	0.375	0.064	5.815	0.000
i	0.692	0.034	20.616	0.000
s	0.135	0.021	6.279	0.000
q	0.002	0.000	4.049	0.000

Multivariate Approach To Multilevel Modeling

Multivariate Modeling Of Family Members

- Multilevel modeling: clusters independent, model for between- and within-cluster variation, units within a cluster statistically equivalent
- Multivariate approach: clusters independent, model for all variables for each cluster unit, different parameters for different cluster units.
  - Used in latent variable growth modeling where the cluster units are the repeated measures over time
  - Allows for different cluster sizes by missing data techniques
  - More flexible than the multilevel approach, but computationally convenient only for applications with small cluster sizes (e.g. twins, spouses)

Figure 1. A Longitudinal Growth Model of Heavy Drinking for Two-Sibling Families



Source: Khoo, S.T. & Muthen, B. (2000). Longitudinal data on families: Growth modeling alternatives. Multivariate Applications in Substance Use Research, J. Rose, L. Chassin, C. Presson & J. Sherman (eds.), Hillsdale, N.J.: Erlbaum, pp. 43-78.

Three-Level Modeling As Single-Level Analysis

Doubly multivariate:

- Repeated measures in wide, multivariate form
- Siblings in wide, multivariate form

It is possible to do four-level by TYPE = TWOLEVEL, for instance families within geographical segments

Input For Multivariate Modeling Of Family Data

```
TITLE:      Multivariate modeling of family data
            one observation per family
DATA:       FILE IS multi.dat;
VARIABLE:   NAMES ARE o18-o32 y18-y32 omale oes ohsdrop ymale yoes
            yhsdrop black hisp fh123 fh1 fh23;
MODEL:      s21o lrateo qrateo | o18@0 o19@1 o20@2 o21@3 o22@4
            o23@5 o24@6 o25@7 o26@8 o27@9 o28@10 o29@11 o30@12
            o31@13 o32@14;
            s21y lratey qratey | y18@0 y19@1 y20@2 y21@3 y22@4 y23@5
            y24@6 y25@7 y26@8 y27@9 y28@10 y29@11 y30@12
            y31@13 y32@14;
            s12o ON omale oes ohsdrop black hisp fh123 fh1 fh23;
            s21y ON male yes yhsdrop black hisp fh123 fh1 fh23;
            s21y ON s21o;
            lratey ON s21o lrateo;
            qratey ON s21o lrateo qrateo;
```


Multilevel Growth Modeling (3-Level Analysis)

Three-Level Analysis In Multilevel Terms

Time point t, individual i, cluster j.

y_{tij} : individual-level, outcome variable
a_{1tij} : individual-level, time-related variable (age, grade)
a_{2tij} : individual-level, time-varying covariate
x_{ij} : individual-level, time-invariant covariate
w_j : cluster-level covariate

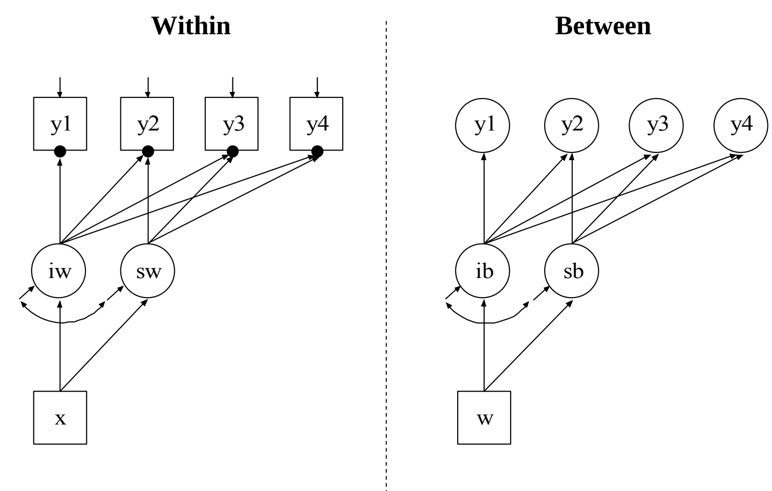
Three-level analysis (Mplus considers Within and Between)

$$Level\ 1\ (Within):\ y_{tij} = \pi_{0ij} + \pi_{1ij}\, a_{1tij} + \pi_{2tij}\, a_{2tij} + e_{tij}, \qquad (1)$$

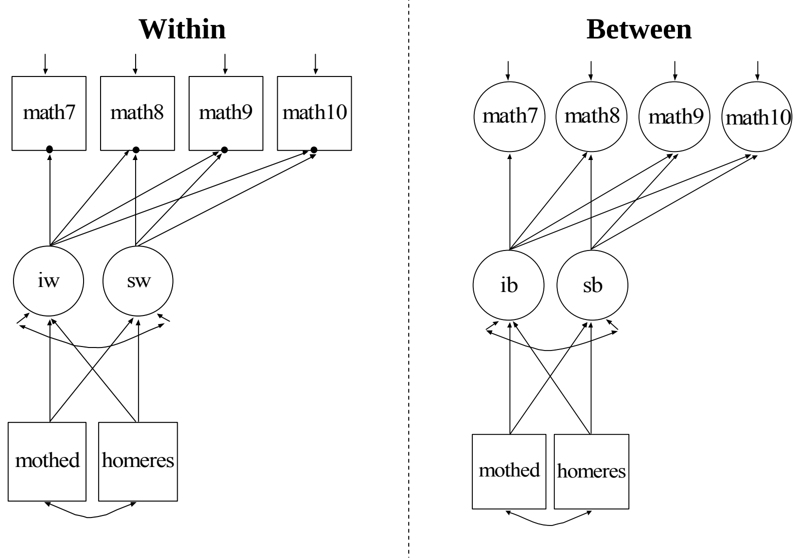
$$Level\ 2\ (Within):\ \begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j}\, x_{ij} + r_{0ij}, & \rightarrow iw \\ \pi_{1ij} = \beta_{10j} + \beta_{11j}\, x_{ij} + r_{1ij}, & \rightarrow sw \\ \pi_{2tij} = \beta_{20tj} + \beta_{21tj}\, x_{ij} + r_{2tij}. \end{cases} \qquad (2)$$

$$Level\ 3\ (Between):\ \begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001}\, w_j + u_{00j}, & \rightarrow ib \\ \beta_{10j} = \gamma_{100} + \gamma_{101}\, w_j + u_{10j}, & \rightarrow sb \\ \beta_{20tj} = \gamma_{200t} + \gamma_{201t}\, w_j + u_{20tj}, \\ \beta_{01j} = \gamma_{010} + \gamma_{011}\, w_j + u_{01j}, \\ \beta_{11j} = \gamma_{110} + \gamma_{111}\, w_j + u_{11j}, \\ \beta_{21tj} = \gamma_{2t0} + \gamma_{2t1}\, w_j + u_{2tj}. \end{cases} \qquad (3)$$

Two-Level Growth Modeling (Three-Level Analysis)



LSAY Two-Level Growth Model



Input For LSAY Two-Level Growth Model With Free Time Scores And Covariates

```
TITLE:      LSAY two-level growth model with free time scores
            and covariates
DATA:       FILE IS lsay98.dat;
            FORMAT IS 3f8 f8.4 8f8.2 3f8 2f8.2;
VARIABLE:   NAMES ARE cohort id school weight math7 math8 math9
            math10 att7 att8 att9 att10 gender mothed homeres;
            USEOBS = (gender EQ 1 AND cohort EQ 2);
            MISSING = ALL (999);
            USEVAR = math7-math10 mothed homeres;
            CLUSTER = school;
ANALYSIS:   TYPE = TWOLEVEL;
            ESTIMATOR = MUML;
```

Input For LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

```
MODEL:      %WITHIN%
            iw sw | math7@0 math8@1
            math9*2  (1)
            math10*3 (2);
            iw sw ON mothed homeres;

            %BETWEEN%
            ib sb | math7@0 math8@1
            math9*2  (1)
            math10*3 (2);
            ib sb ON mothed homeres;

OUTPUT      SAMPSTAT STANDARDIZED RESIDUAL;
```


Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates

Summary of Data

```
  Number of clusters       50

    Size (s)  Cluster ID with Size s

      1         114
      2         136
      6         132     304

     34         104
     39         309
     40         302

  Average cluster size 18.627

  Estimated Intraclass Correlations for the Y Variables
               Intraclass              Intraclass             Intraclass
Variable      Correlation  Variable   Correlation  Variable   Correlation
MATH7            0.199     MATH8         0.149     MATH9          0.168
MATH10           0.165
```

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Tests Of Model Fit

```
Chi-square Test of Model Fit
            Value                           24.058*
            Degrees of Freedom                  14
            P-Value                          0.0451
CFI / TLI
            CFI                              0.997
            TLI                              0.995
RMSEA (Root Mean Square Error Of Approximation)
            Estimate                         0.028
SRMR (Standardized Root Mean Square Residual)
            Value for Between                0.048
            Value for Within                 0.007
```


Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Model Results

Within Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
SW |					
MATH8	1.000	0.000	0.000	1.073	0.128
MATH9	2.487	0.163	15.220	2.670	0.288
MATH10	3.589	0.223	16.076	3.853	0.368
IW ON					
MOTHED	**1.780**	**0.232**	**7.665**	**0.246**	**0.226**
HOMERES	**0.892**	**0.221**	**4.031**	**0.124**	**0.173**
SW ON					
MOTHED	**0.053**	**0.063**	**0.836**	**0.049**	**0.045**
HOMERES	**0.135**	**0.044**	**3.047**	**0.125**	**0.176**

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
SW WITH					
IW	2.112	0.522	4.044	0.273	0.273
HOMERES WITH					
MOTHED	0.261	0.039	6.709	0.261	0.203
Residual Variances					
MATH7	12.748	1.434	8.888	12.748	0.197
MATH8	12.298	0.893	13.771	12.298	0.174
MATH9	14.237	1.132	12.578	14.237	0.166
MATH10	24.829	2.230	11.133	24.829	0.226
IW	47.060	3.069	15.333	0.903	0.903
SW	1.110	0.286	3.879	0.964	0.964
Variances					
MOTHED	0.841	0.049	17.217	0.841	1.000
HOMERES	1.970	0.069	28.643	1.970	1.000

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Between Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
SB |					
MATH8	1.000	0.000	0.000	0.196	0.052
MATH9	2.487	0.163	15.220	0.488	0.119
MATH10	3.589	0.223	16.076	0.704	0.115
IB ON					
MOTHED	**-1.225**	**2.587**	**-0.474**	**-0.362**	**-0.107**
HOMERES	**7.160**	**1.847**	**3.876**	**2.117**	**1.011**
SB ON					
MOTHED	**0.995**	**0.647**	**1.538**	**5.073**	**1.493**
HOMERES	**0.017**	**0.373**	**0.045**	**0.086**	**0.041**
SB WITH					
IB	0.382	0.248	1.538	0.575	0.575

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

HOMERES WITH					
MOTHED	0.103	0.019	5.488	0.103	0.733
Residual Variances					
MATH7	2.059	0.552	3.732	2.059	0.153
MATH8	0.544	0.268	2.033	0.544	0.039
MATH9	0.105	0.213	0.493	0.105	0.006
MATH10	1.395	0.504	2.767	1.395	0.067
IB	1.428	1.690	0.845	0.125	0.125
SB	-0.051	0.071	-0.713	-1.321	-1.321
Variances					
MOTHED	0.087	0.023	3.801	0.087	1.000
HOMERES	0.228	0.056	4.066	0.228	1.000
Means					
MOTHED	2.307	0.043	53.277	2.307	7.838
HOMERES	3.108	0.062	50.375	3.108	6.509
Intercepts					
IB	33.510	2.678	12.512	9.909	9.909
SB	0.163	0.776	0.210	0.830	0.830

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

R-Square

Within Level

Observed Variable	R-Square
MATH7	0.803
MATH8	0.826
MATH9	0.834
MATH10	0.774

Latent Variable	R-Square
IW	0.097
SW	0.036

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

R-Square

```
Between Level

    Observed
    Variable    R-Square

    MATH7       0.847
    MATH8       0.961
    MATH9       0.994
    MATH10      0.933

    Latent
    Variable    R-Square

    IB          0.875
    SB          Undefined   0.23207E+01
```


Further Readings On Three-Level Growth Analysis

Muthén, B. (1997). Latent variable modeling with longitudinal and multilevel data. In A. Raftery (ed), Sociological Methodology (pp. 453-480). Boston: Blackwell Publishers. (#73)

Muthén, B & Asparouhov, T. (2009). Beyond multilevel regression modeling: Multilevel analysis in a general latent variable framework. To appear in The Handbook of Advanced Multilevel Analysis. J. Hox & J.K. Roberts (eds). Taylor and Francis.

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

Multilevel Growth Modeling Of Binary Outcomes (3-Level Analysis)

Multilevel Growth Model With Binary Outcomes

Time point t, individual i, cluster j.

Logit-linear growth.

Level 1 (*Within Cluster*):

$$log\left[\frac{P(u_{ti}=1 \mid \eta_{0ij},\eta_{1ij},a_{ti})}{P(u_{ti}=0 \mid \eta_{0ij},\eta_{1ij},a_{ti})}\right]=\eta_{0ij}+\eta_{1ij}\cdot(a_{ti}-c) \quad (1)$$

Level 2 (*Within Cluster*):

$$\begin{cases}\eta_{0ij}=\beta_{00j}+\underbrace{\beta_{01}\,x_{ij}+r_{0ij}}_{\to\ \eta_{0w}},\\ \eta_{1ij}=\beta_{10j}+\underbrace{\beta_{11}\,x_{ij}+r_{1ij}}_{\to\ \eta_{1w}},\end{cases} \quad (2)$$

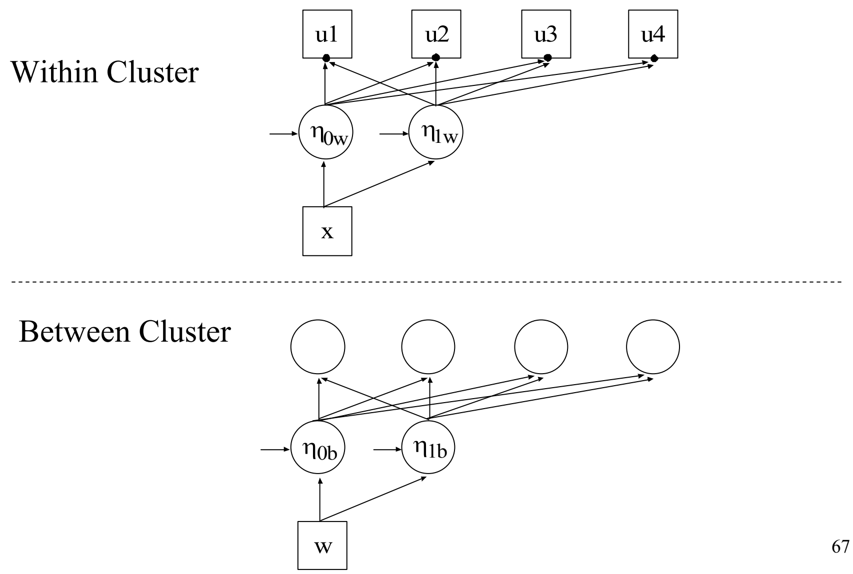
Level 3 (*Between Cluster*):

$$\begin{cases}\beta_{00j}=\underbrace{\gamma_{000}+\gamma_{001}\,w_j+r_{00j}}_{\to\ \eta_{0b}},\\ \beta_{10j}=\underbrace{\gamma_{100}+\gamma_{101}\,w_j+r_{10j}}_{\to\ \eta_{1b}},\end{cases} \quad (3)$$

Fitzmaurice, Laird, & Ware (2004). Applied longitudinal analysis. Wiley & Sons.

Fitzmaurice, Davidian, Verbeke, & Molenberghs (2008). Longitudinal Data Analysis. Chapman & Hall/CRC Press.

Growth Model With Binary Outcomes



Observed Data Likelihood

Individual *i* in cluster j

$$\prod_j \int \phi_j(\eta_{bj}) \prod_i \left(\int f_{ij}(u_{ij} \mid \eta_{bj}, \eta_{wij}) \phi_{ij}(\eta_{wij}) \, d\eta_{wij} \right) d\eta_{bj}$$

- Maximum likelihood estimation
- Numerical integration
- The logit-linear binary growth model results in 4 dimensions of integration

Input Excerpts Two-Level Binary Growth

```
          CLUSTER = classrm;
          USEVAR = bkthin1f bkthin1s bkthin2s bkthin3s bkthin4s
          bkthin5s bkthin6s bkthin7s male;
          CATEGORICAL = bkthin1f - bkthin7s;
          WITHIN = male;
          MISSING = ALL (999);
DEFINE:   CUT bkThin1f(1.5);
          CUT bkThin1s(1.5);
          CUT bkThin2s(1.5);
          CUT bkThin3s(1.5);
          CUT bkThin4s(1.5);
          CUT bkThin5s(1.5);
          CUT bkThin6s(1.5);
          CUT bkThin7s(1.5);
          male = 2 - gender;
ANALYSIS: TYPE = TWOLEVEL;
          PROCESS = 4;
          ESTIMATOR = WLSM;
```


Input Excerpts Two-Level Binary Growth

```
MODEL:     %WITHIN%

           iw sw qw | bkthin1f@0 bkthin1s@.5 bkthin2s@1.5
           bkthin3s@2.5 bkthin4s@3.5 bkthin5s@4.5 bkthin6s@5.5
           bkthin7s@6.5;

           iw-qw on male;

           %BETWEEN%

           ib sb qb |bkthin1f@0 bkthin1s@.5 bkthin2s@1.5
           bkthin3s@2.5 bkthin4s@3.5 bkthin5s@4.5 bkthin6s@5.5
           bkthin7s@6.5;

           bkthin1f-bkthin7s@0;


OUTPUT:    TECH1 TECH8 STANDARDIZED;
```

Output Excerpts Two-Level Binary Growth

```
Number of clusters                      41
Average cluster size                28.634
```

Estimated Intraclass Correlations for the Y Variables

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
bkthin1f	0.470	bkthin1s	0.484	bkthin2s	0.385
bkthin3s	0.089	bkthin4s	0.052	bkthin5s	0.094
bkthin6s	0.137	bkthin7s	0.104		

Output Excerpts Two-Level Binary Growth (Continued)

```
Tests of Model Fit

Chi-Square Test of Model Fit
     Value                                  101.742*
     Degrees of Freedom                           62
     P-Value                                  0.0011

*    The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM
     and WLSMV cannot be used for chi-square difference
     tests. MLM, MLR and WLSM chi-square difference testing
     is described in the Mplus Technical Appendices at
     www.statmodel.com. See chi-square difference testing in
     the index of the Mplus Users' Guide.
```

Output Excerpts Two-Level Binary Growth (Continued)

```
Chi-Square Test of Model Fit for the Baseline Model
     Value                                        924.374
     Degrees of Freedom                                64
     P-Value                                       0.0000
CFI/TLI
     CFI                                            0.954
     TLI                                            0.952
Number of Free Parameters                              18
RMSEA (Root Mean Square Error Of Approximation)
     Estimate                                       0.023
SRMR (Standardized Root Mean Square Residual)
     Value for Within                               0.056
     Value for Between                              0.513
```


Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
iw |				
bkthin1f	1.000	0.000	999.000	999.000
bkthin1s	1.000	0.000	999.000	999.000
bkthin2s	1.000	0.000	999.000	999.000
bkthin3s	1.000	0.000	999.000	999.000
bkthin4s	1.000	0.000	999.000	999.000
bkthin5s	1.000	0.000	999.000	999.000
bkthin6s	1.000	0.000	999.000	999.000
bkthin7s	1.000	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
sw |				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.500	0.000	999.000	999.000
bkthin2s	1.500	0.000	999.000	999.000
bkthin3s	2.500	0.000	999.000	999.000
bkthin4s	3.500	0.000	999.000	999.000
bkthin5s	4.500	0.000	999.000	999.000
bkthin6s	5.500	0.000	999.000	999.000
bkthin7s	6.500	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
qw |				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.250	0.000	999.000	999.000
bkthin2s	2.250	0.000	999.000	999.000
bkthin3s	6.250	0.000	999.000	999.000
bkthin4s	12.250	0.000	999.000	999.000
bkthin5s	20.250	0.000	999.000	999.000
bkthin6s	30.250	0.000	999.000	999.000
bkthin7s	42.250	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
iw ON				
male	0.980	0.131	7.496	0.000
sw ON				
male	-0.177	0.087	-2.032	0.042
qw ON				
male	0.023	0.013	1.707	0.088
sw WITH				
iw	-0.290	0.121	-2.388	0.017
qw WITH				
iw	0.022	0.016	1.434	0.151
sw	-0.010	0.010	-0.987	0.324

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Residual Variances				
iw	1.315	0.284	4.637	0.000
sw	0.102	0.071	1.430	0.153
qw	0.001	0.001	0.683	0.495
Between Level				
ib |				
bkthin1f	1.000	0.000	999.000	999.000
bkthin1s	1.000	0.000	999.000	999.000
bkthin2s	1.000	0.000	999.000	999.000
bkthin3s	1.000	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
bkthin4s	1.000	0.000	999.000	999.000
bkthin5s	1.000	0.000	999.000	999.000
bkthin6s	1.000	0.000	999.000	999.000
bkthin7s	1.000	0.000	999.000	999.000
sb |				
bkthin1f	1.000	0.000	999.000	999.000
bkthin1s	0.500	0.000	999.000	999.000
bkthin2s	1.500	0.000	999.000	999.000
bkthin3s	2.500	0.000	999.000	999.000
bkthin4s	3.500	0.000	999.000	999.000
bkthin5s	4.500	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
bkthin6s	5.500	0.000	999.000	999.000
bkthin7s	6.500	0.000	999.000	999.000
qb |				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.250	0.000	999.000	999.000
bkthin2s	2.250	0.000	999.000	999.000
bkthin3s	6.250	0.000	999.000	999.000
bkthin4s	12.250	0.000	999.000	999.000
bkthin5s	20.250	0.000	999.000	999.000
bkthin6s	30.250	0.000	999.000	999.000
bkthin7s	42.250	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
sw WITH				
ib	-0.395	0.122	-3.246	0.001
qb WITH				
ib	0.043	0.014	3.069	0.002
sb	-0.018	0.007	-2.451	0.014
Means				
ib	0.000	0.000	999.000	999.000
sb	0.219	0.133	1.648	0.099
qb	-0.028	0.018	-1.614	0.107

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Thresholds				
bkthin1f$1	1.592	0.253	6.304	0.000
bkthin1s$1	1.592	0.253	6.304	0.000
bkthin2s$1	1.592	0.253	6.304	0.000
bkthin3s$1	1.592	0.253	6.304	0.000
bkthin4s$1	1.592	0.253	6.304	0.000
bkthin5s$1	1.592	0.253	6.304	0.000
bkthin6s$1	1.592	0.253	6.304	0.000
bkthin7s$1	1.592	0.253	6.304	0.000

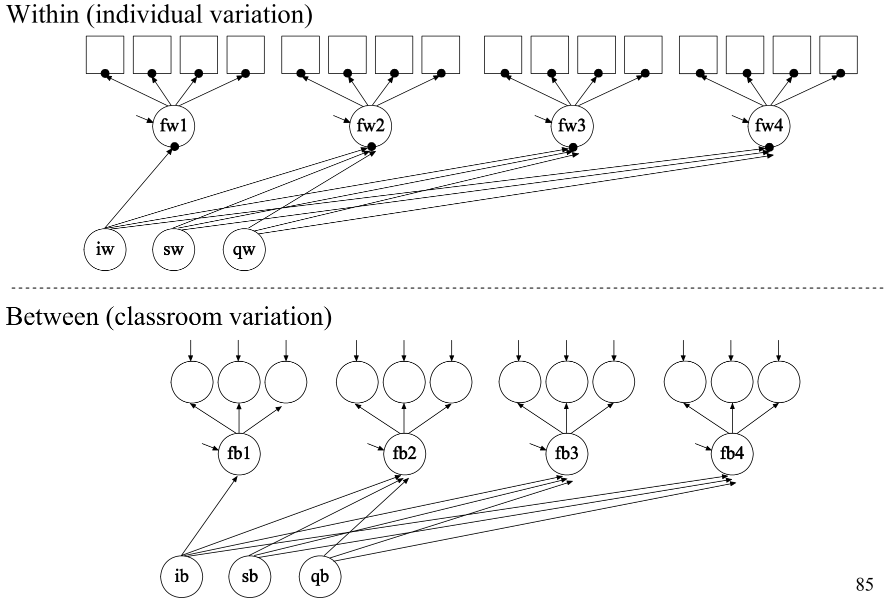
Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Variances				
ib	0.935	0.285	3.278	0.001
sb	0.165	0.058	2.842	0.004
qb	0.002	0.001	2.071	0.038
Residual Variances				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.000	0.000	999.000	999.000
bkthin2s	0.000	0.000	999.000	999.000

Output Excerpts Two-Level Binary Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
bkthin3s	0.000	0.000	999.000	999.000
bkthin4s	0.000	0.000	999.000	999.000
bkthin5s	0.000	0.000	999.000	999.000
bkthin6s	0.000	0.000	999.000	999.000
bkthin7s	0.000	0.000	999.000	999.000

Two-Level Multiple Indicator Growth



Multilevel Multiple Indicator Growth In Other Software

Treated as 4-level analysis:

- Level 1: Indicators
- Level 2: Time points
- Level 3: Individuals
- Level 4: Clusters

This approach is

- Cumbersome requiring dummy variables to indicate indicators
- Limiting in that only intercepts/thresholds and not loadings are allowed to vary across indicators (Rasch model for categorical outcomes).

Multilevel Multiple Indicator Growth In Other Software (Continued)

In contrast, Mplus treats such analysis as two-level modeling with

- Within: Indicators, time points, individuals (doubly wide format: indicators x time points)
- Between: Clusters

  and allowing intercepts/thresholds and loadings to vary across indicators and, if needed, across time.

Input Excerpts Two-Level Multiple Indicator Growth

```
ANALYSIS:   TYPE = TWOLEVEL;
            PROCESS = 4;
            ESTIMATOR = WLSM;
MODEL:      %WITHIN%
            f1 BY bkthin1f
            harmo1f (1)
            takep1f (2);
            f2 BY bkthin1s
            harmo1s (1)
            takep1s (2);
            f3 BY bkthin2s
            harmo2s (1)
            takep2s (2);
```

Input Excerpts Two-Level Multiple Indicator Growth (Continued)

```
f4 BY bkthin3s
harmo3s (1)
takep3s (2);
f5 BY bkthin4s
harmo4s (1)
takep4s (2);
f6 BY bkthin5s
harmo5s (1)
takep5s (2);
f7 BY bkthin6s
harmo6s (1)
takep6s (2);
```


Input Excerpts Two-Level Multiple Indicator Growth (Continued)

```
f8 BY bkthin7s
harmo7s (1)
takep7s (2);
iw sw qw | f1@0 f2@.5 f3@1.5 f4@2.5 f5@3.5 f6@4.5
f7@5.5 f8@6.5;
iw-qw ON male;
%BETWEEN%
f1b BY bkthin1f
harmo1f (3)
takep1f (4);
f2b BY bkthin1s
harmo1s (3)
takep1s (4);
```

Input Excerpts Two-Level Multiple Indicator Growth (Continued)

```
f3b BY bkthin2s
harmo2s (3)
takep2s (4);
f4b BY bkthin3s
harmo3s (3)
takep3s (4);
f5b BY bkthin4s
harmo4s (3)
takep4s (4);
f6b BY bkthin5s
harmo5s (3)
takep5s (4);
```


Input Excerpts Two-Level Multiple Indicator Growth (Continued)

```
            f7b BY bkthin6s
            harmo6s (3)
            takep6s (4);
            f8b BY bkthin7s
            harmo7s (3)
            takep7s (4);
            [bkthin1f$1-bkthin7s$1] (11);
            [harmo1f$1-harmo7s$1] (12);
            [takep1f$1-takep7s$1] (13);
            ib sb qb |f1b@0 f2b@.5 f3b@1.5 f4b@2.5 f5b@3.5 f6b@4.5
            f7b@5.5 f8b@6.5;
            bkthin1f-takep7s@0;
OUTPUT:     TECH1 TECH8 STANDARDIZED;
```

Output Excerpts Two-Level Multiple Indicator Growth

```
Tests of Model Fit

Chi-Square Test of Model Fit
     Value                                      767.730*
     Degrees of Freedom                              584
     P-Value                                      0.0000

*    The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and
     WLSMV cannot be used for chi-square difference tests. MLM,
     MLR and WLSM chi-square difference testing is described in
     the Mplus Technical Appendices at www.statmodel.com. See chi-
     square difference testing in the index of the Mplus Users'
     Guide.
```


Output Excerpts Two-Level Multiple Indicator Growth (Continued)

```
Chi-Square Test of Model Fit for the Baseline Model
     Value                                     28765.253
     Degrees of Freedom                              576
     P-Value                                      0.0000
CFI/TLI
     CFI                                           0.993
     TLI                                           0.994
Number of Free Parameters                             40
RMSEA (Root Mean Square Error of Approximation)
     Estimate                                      0.016
SRMR (Standardized Root Mean Square Residual)
     Value for Within                              0.065
     Value for Between                             0.232
```

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
iw |				
f1	1.000	0.000	999.000	999.000
f2	1.000	0.000	999.000	999.000
f3	1.000	0.000	999.000	999.000
f4	1.000	0.000	999.000	999.000
f5	1.000	0.000	999.000	999.000
f6	1.000	0.000	999.000	999.000
f7	1.000	0.000	999.000	999.000
f8	1.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
sw |				
f1	0.000	0.000	999.000	999.000
f2	0.500	0.000	999.000	999.000
f3	1.500	0.000	999.000	999.000
f4	2.500	0.000	999.000	999.000
f5	3.500	0.000	999.000	999.000
f6	4.500	0.000	999.000	999.000
f7	5.500	0.000	999.000	999.000
f8	6.500	0.000	999.000	999.000
qw |				
f1	0.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f2	0.250	0.000	999.000	999.000
f3	2.250	0.000	999.000	999.000
f4	6.250	0.000	999.000	999.000
f5	12.250	0.000	999.000	999.000
f6	20.250	0.000	999.000	999.000
f7	30.250	0.000	999.000	999.000
f8	42.250	0.000	999.000	999.000
f1 BY				
bkthin1f	1.000	0.000	999.000	999.000
harmo1f	1.143	0.088	12.932	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
harmo3s	1.143	0.088	12.932	0.000
takep3s	0.993	0.067	14.782	0.000
f5 BY				
bkthin4s	1.000	0.000	999.000	999.000
harmo4s	1.143	0.088	12.932	0.000
takep4s	0.993	0.067	14.782	0.000
f6 BY				
bkthin5s	1.000	0.000	999.000	999.000
harmo5s	1.143	0.088	12.932	0.000
takep5s	0.993	0.067	14.782	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f7 BY				
bkthin6s	1.000	0.000	999.000	999.000
harmo6s	1.143	0.088	12.932	0.000
takep6s	0.993	0.067	14.782	0.000
f8 BY				
bkthin7s	1.000	0.000	999.000	999.000
harmo7s	1.143	0.088	12.932	0.000
takep7s	0.993	0.067	14.782	0.000
iw ON				
male	1.449	0.175	8.258	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
sw ON				
male	-0.091	0.129	-0.705	0.481
qw ON				
male	0.011	0.021	0.553	0.580
sw WITH				
iw	-0.394	0.141	-2.793	0.005
qw WITH				
iw	0.017	0.020	0.835	0.404
sw	-0.031	0.016	-1.973	0.049
Residual Variances				
f1	1.093	0.259	4.221	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f2	0.739	0.236	3.131	0.002
f3	2.850	0.497	5.731	0.000
f4	1.985	0.561	3.541	0.000
f5	2.255	0.487	4.630	0.000
f6	2.581	0.317	8.150	0.000
f7	2.171	0.383	5.667	0.000
f8	2.332	0.557	4.188	0.000
iw	2.812	0.366	7.679	0.000
sw	0.277	0.110	2.510	0.012
qw	0.004	0.002	1.797	0.072

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Between Level				
ib |				
f1b	1.000	0.000	999.000	999.000
f2b	1.000	0.000	999.000	999.000
f3b	1.000	0.000	999.000	999.000
f4b	1.000	0.000	999.000	999.000
f5b	1.000	0.000	999.000	999.000
f6b	1.000	0.000	999.000	999.000
f7b	1.000	0.000	999.000	999.000
f8b	1.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
sb |				
f1b	0.000	0.000	999.000	999.000
f2b	0.500	0.000	999.000	999.000
f3b	1.500	0.000	999.000	999.000
f4b	2.500	0.000	999.000	999.000
f5b	3.500	0.000	999.000	999.000
f6b	4.500	0.000	999.000	999.000
f7b	5.500	0.000	999.000	999.000
f8b	6.500	0.000	999.000	999.000
qb |				
f1b	0.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f2b	0.250	0.000	999.000	999.000
f3b	2.250	0.000	999.000	999.000
f4b	6.250	0.000	999.000	999.000
f5b	12.250	0.000	999.000	999.000
f6b	20.250	0.000	999.000	999.000
f7b	30.250	0.000	999.000	999.000
f8b	42.250	0.000	999.000	999.000
f1b BY				
bkthin1f	1.000	0.000	999.000	999.000
harmo1f	1.099	0.145	7.603	0.000
takep1f	0.979	0.126	7.798	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f2b BY				
bkthin1s	1.000	0.000	999.000	999.000
harmo1s	1.099	0.145	7.603	0.000
takep1s	0.979	0.126	7.798	0.000
f3b BY				
bkthin2s	1.000	0.000	999.000	999.000
harmo2s	1.099	0.145	7.603	0.000
takep2s	0.979	0.126	7.798	0.000
f4b BY				
bkthin3s	1.000	0.000	999.000	999.000
harmo3s	1.099	0.145	7.603	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
takep3s	0.979	0.126	7.798	0.000
f5b BY				
bkthin4s	1.000	0.000	999.000	999.000
harmo4s	1.099	0.145	7.603	0.000
takep4s	0.979	0.126	7.798	0.000
f6b BY				
bkthin5s	1.000	0.000	999.000	999.000
harmo5s	1.099	0.145	7.603	0.000
takep5s	0.979	0.126	7.798	0.000
f7b BY				
bkthin6s	1.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
harmo6s	1.099	0.145	7.603	0.000
takep6s	0.979	0.126	7.798	0.000
f8b BY				
bkthin7s	1.000	0.000	999.000	999.000
harmo7s	1.099	0.145	7.603	0.000
takep7s	0.979	0.126	7.798	0.000
sb WITH				
ib	-0.563	0.200	-2.813	0.005
qb WITH				
ib	0.066	0.023	2.927	0.003
sb	-0.031	0.012	-2.530	0.011

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means				
f1	0.000	0.000	999.000	999.000
f2	0.000	0.000	999.000	999.000
f3	0.000	0.000	999.000	999.000
f4	0.000	0.000	999.000	999.000
f5	0.000	0.000	999.000	999.000
f6	0.000	0.000	999.000	999.000
f7	0.000	0.000	999.000	999.000
f8	0.000	0.000	999.000	999.000
ib	0.000	0.000	999.000	999.000
sb	0.063	0.170	0.371	0.711

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
qb	-0.014	0.023	-0.615	0.538
Intercepts				
f1b	0.000	0.000	999.000	999.000
f2b	0.000	0.000	999.000	999.000
f3b	0.000	0.000	999.000	999.000
f4b	0.000	0.000	999.000	999.000
f5b	0.000	0.000	999.000	999.000
f6b	0.000	0.000	999.000	999.000
f7b	0.000	0.000	999.000	999.000
f8b	0.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Thresholds				
bkthin1f$1	2.364	0.357	6.625	0.000
bkthin1s$1	2.364	0.357	6.625	0.000
bkthin2s$1	2.364	0.357	6.625	0.000
bkthin3s$1	2.364	0.357	6.625	0.000
bkthin4s$1	2.364	0.357	6.625	0.000
bkthin5s$1	2.364	0.357	6.625	0.000
bkthin6s$1	2.364	0.357	6.625	0.000
bkthin7s$1	2.364	0.357	6.625	0.000
harmo1f$1	1.676	0.353	4.746	0.000
harmo1s$1	1.676	0.353	4.746	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
harmo2s$1	1.676	0.353	4.746	0.000
harmo3s$1	1.676	0.353	4.746	0.000
harmo4s$1	1.676	0.353	4.746	0.000
harmo5s$1	1.676	0.353	4.746	0.000
harmo6s$1	1.676	0.353	4.746	0.000
harmo7s$1	1.676	0.353	4.746	0.000
takep1f$1	1.612	0.288	5.595	0.000
takep1s$1	1.612	0.288	5.595	0.000
takep2s$1	1.612	0.288	5.595	0.000
takep3s$1	1.612	0.288	5.595	0.000
takep4s$1	1.612	0.288	5.595	0.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
takep5s$1	1.612	0.288	5.595	0.000
takep6s$1	1.612	0.288	5.595	0.000
takep7s$1	1.612	0.288	5.595	0.000
Variances				
ib	1.328	0.515	2.580	0.010
sb	0.258	0.097	2.669	0.008
qb	0.004	0.002	2.298	0.022
Residual Variances				
bkthin1f	0.000	0.000	999.000	999.000
bkthin1s	0.000	0.000	999.000	999.000
bkthin2s	0.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
bkthin3s	0.000	0.000	999.000	999.000
bkthin4s	0.000	0.000	999.000	999.000
bkthin5s	0.000	0.000	999.000	999.000
bkthin6s	0.000	0.000	999.000	999.000
bkthin7s	0.000	0.000	999.000	999.000
harmo1f	0.000	0.000	999.000	999.000
harmo1s	0.000	0.000	999.000	999.000
harmo2s	0.000	0.000	999.000	999.000
harmo3s	0.000	0.000	999.000	999.000
harmo4s	0.000	0.000	999.000	999.000
harmo5s	0.000	0.000	999.000	999.000

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

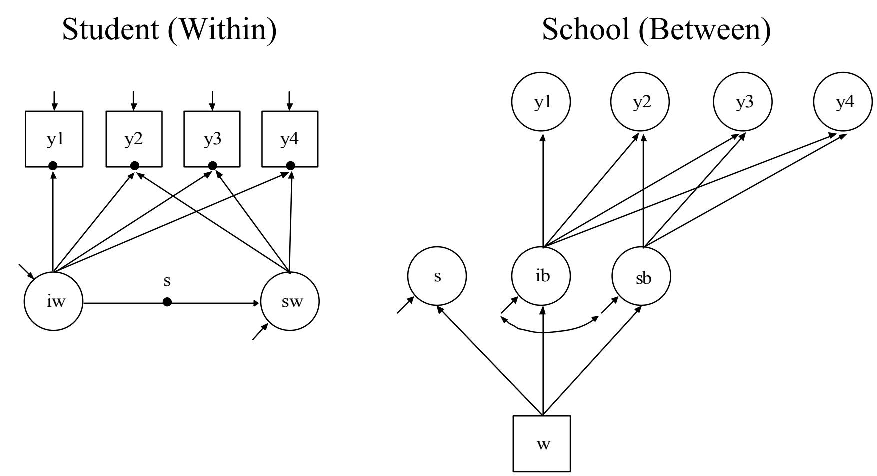
Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
harmo6s	0.000	0.000	999.000	999.000
harmo7s	0.000	0.000	999.000	999.000
takep1f	0.000	0.000	999.000	999.000
takep1s	0.000	0.000	999.000	999.000
takep2s	0.000	0.000	999.000	999.000
takep3s	0.000	0.000	999.000	999.000
takep4s	0.000	0.000	999.000	999.000
takep5s	0.000	0.000	999.000	999.000
takep6s	0.000	0.000	999.000	999.000
takep7s	0.000	0.000	999.000	999.000
f1b	2.258	0.786	2.871	0.004

Output Excerpts Two-Level Multiple Indicator Growth (Continued)

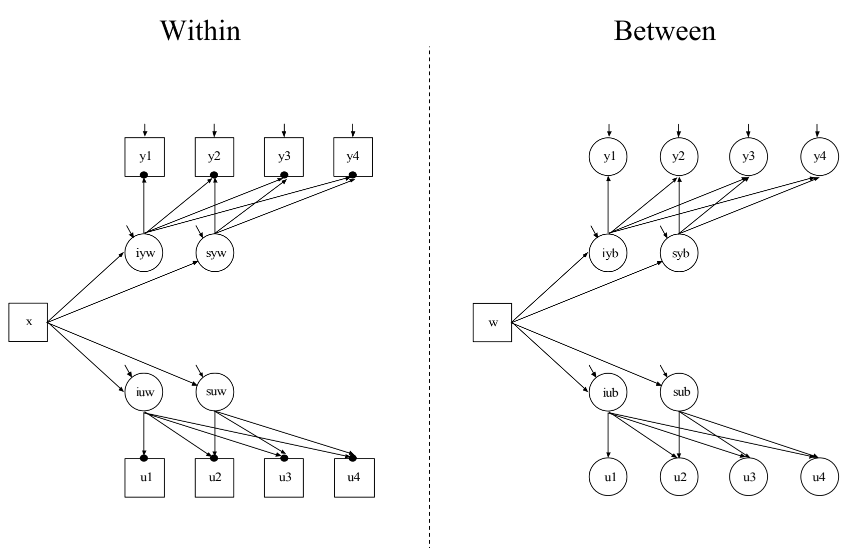
Parameter	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
f2b	2.206	0.821	2.685	0.007
f3b	3.002	1.062	2.827	0.005
f4b	0.377	0.237	1.594	0.111
f5b	0.051	0.117	0.440	0.660
f6b	0.251	0.214	1.176	0.239
f7b	0.543	0.303	1.790	0.073
f8b	-0.468	0.298	-1.567	0.117

Special Multilevel Growth Models

Multilevel Modeling With A Random Slope For Latent Variables

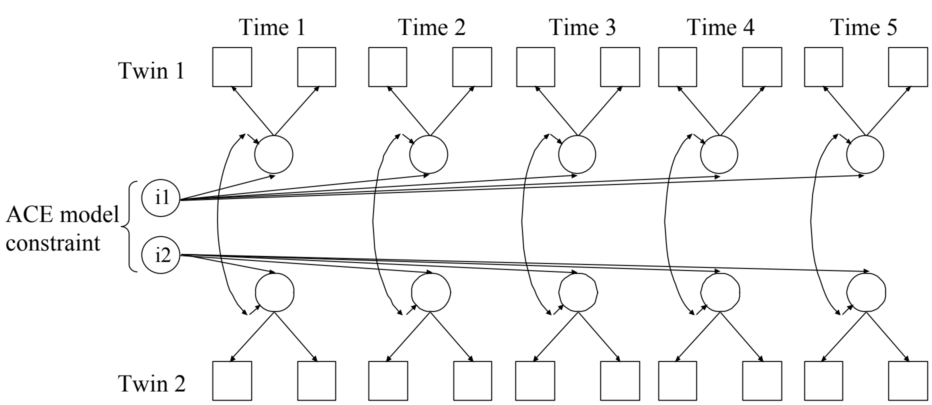


Two-Level, Two-Part Growth Modeling



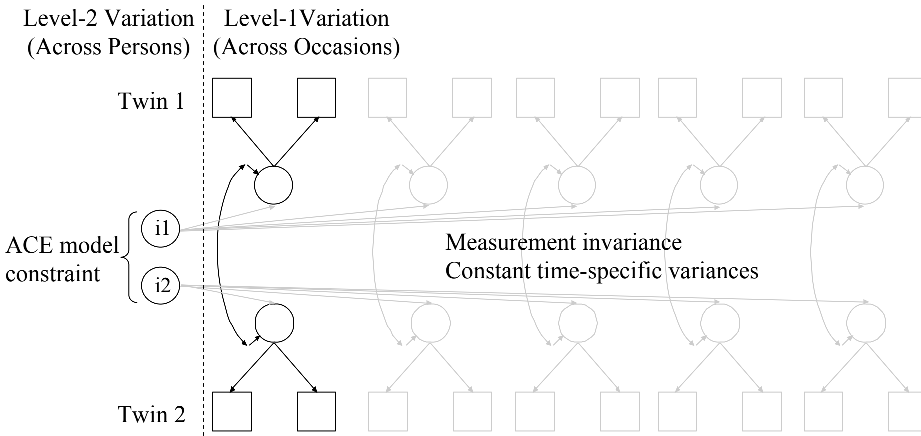
Multiple Indicator Growth Modeling As Two-Level Analysis

Wide Data Format, Single-Level Approach



20 variables, 12 factors, 10 dimensions of integration for ML with categorical items
ML very hard, WLS easy

Long Format, Two-Level Approach



4 variables, 2 Level-2 and 2 Level-1 factors, 4 dimensions of integration for ML with categorical items

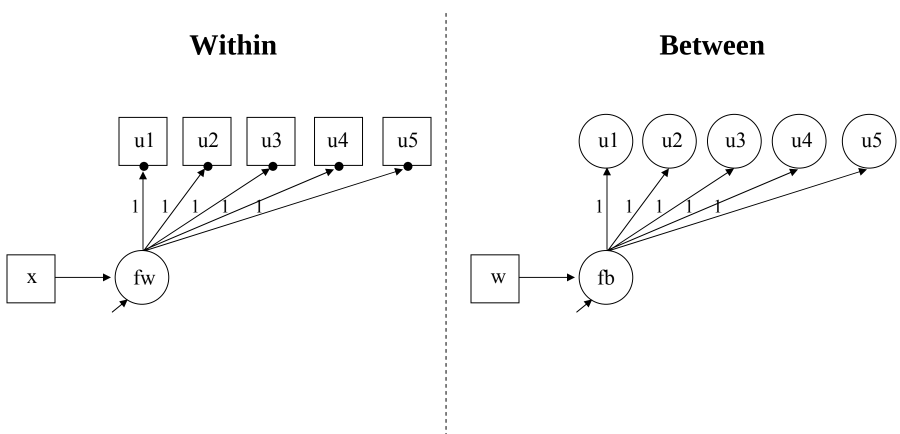
ML feasible, WLS available in Version 5

Multilevel Discrete-Time Survival Analysis

Multilevel Discrete-Time Survival Analysis

- Muthén and Masyn (2005) in Journal of Educational and Behavioral Statistics
- Masyn dissertation
- Continuous-time survival: Asparouhov, Masyn and Muthén (2006)

Multilevel Discrete-Time Survival Frailty Modeling



Vermunt (2003)

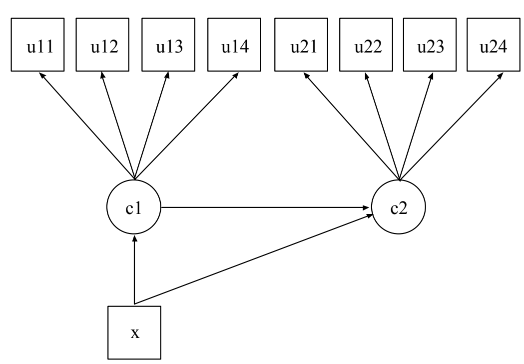
Two-Level Latent Transition Analysis

Latent Transition Analysis

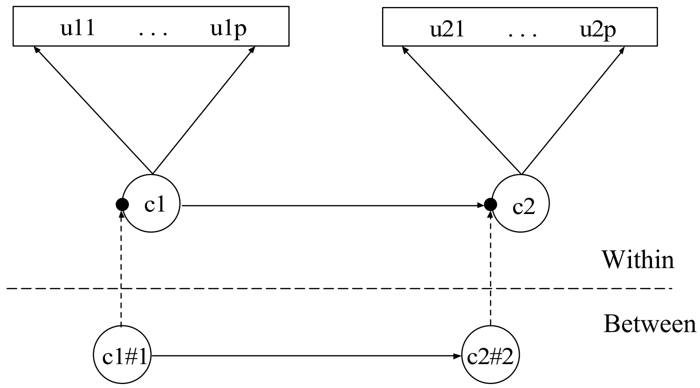
Transition Probabilities

c1 \ c2	1	2
1	0.8	0.2
2	0.4	0.6

Time Point 1 Time Point 2



Two-Level Latent Transition Analysis



Asparouhov, T. & Muthen, B. (2008). Multilevel mixture models. In Hancock, G. R., & Samuelsen, K. M. (Eds.). Advances in latent variable mixture models, pp 27 - 51. Charlotte, NC: Information Age Publishing, Inc.

Input For Two-Level LTA

```
         CLUSTER = classrm;
         USEVAR = stub1f bkrule1f bkthin1f-tease1f athort1f
                   stub1s bkrule1s bkthin1s-tease1s athort1s;
         CATEGORICAL =  stub1f-athort1s;
         MISSING = all (999);
         CLASSES = cf(2) cs(2);
DEFINE:
         CUT stub1f-athort1s(1.5);
ANALYSIS:
         TYPE = TWOLEVEL MIXTURE;
         PROCESS = 2;
MODEL:
         %WITHIN%
         %OVERALL%
         cs#1 ON cf#1;
         %BETWEEN%
         OVERALL%
         cs#1 ON cf#1;
         cs#1*1 cf#1*1;
```

Input For Two-Level LTA (Continued)

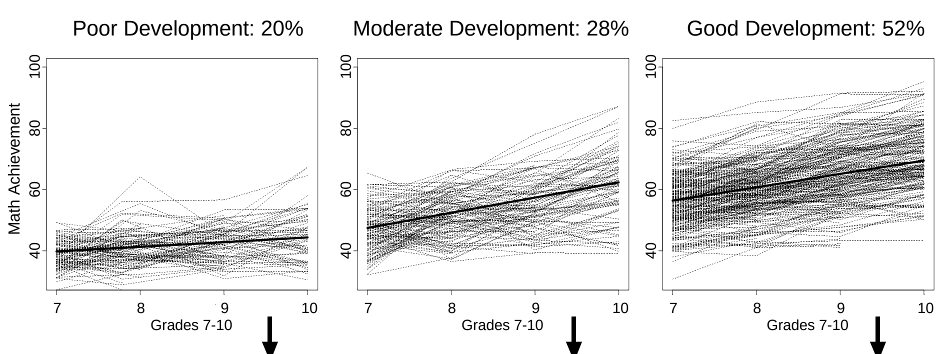
```
MODEL cf:
   %BETWEEN%
        %cf#1%
        [stub1f$1-athort1f$1] (1-9);
        %cf#2%
        [stub1f$1-athort1f$1] (11-19);
MODEL cs:
   %BETWEEN%
        %cs#1%
        [stub1s$1-athort1s$1] (1-9);
        %cs#2%
        [stub1s$1-athort1s$1] (11-19);
OUTPUT:
        TECH1 TECH8;
PLOT:
        TYPE = PLOT3;
        SERIES = stub1f-athort1f(*);
```


Output Excerpts Two-Level LTA

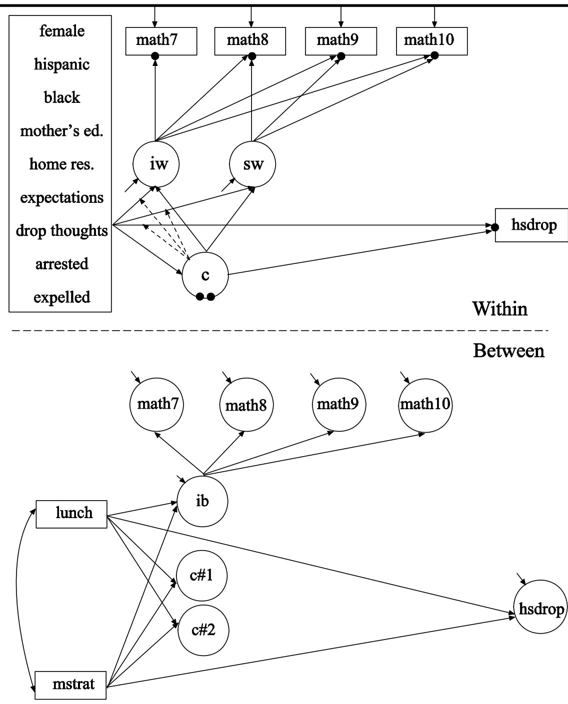
```
Categorical Latent Variables
Within Level
                            Estimates     S.E.    Est./S.E.
  CS#1      ON
      CF#1                     3.938      0.407      9.669
Means
      CF#1                    -0.126      0.189     -0.664
      CS#1                    -1.514      0.221     -6.838
Between Level
  CS#1      ON
      CF#1                     0.411       0.15      2.735
Variances
      CF#1                     2.062      0.672      3.067
Residual Variances
      CS#1                     0.469      0.237      1.984
```

Multilevel Growth Mixture Modeling

Growth Mixture Modeling: LSAY Math Achievement Trajectory Classes And The Prediction Of High School Dropout



Dropout:	69%	8%	1%



Input For A Multilevel Growth Mixture Model For LSAY Math Achievement

```
TITLE:      multilevel growth mixture model for LSAY math
            achievement

DATA:       FILE = lsayfull_Dropout.dat;

VARIABLE:   NAMES = female mothed homeres math7 math8 math9 math10
            expel arrest hisp black hsdrop expect lunch mstrat
            droptht7 schcode;
            !lunch = % of students eligible for full lunch
            !assistance (9th)
            !mstrat = ratio of students to full time math
            !teachers (9th)
            MISSING = ALL (9999);
            CATEGORICAL = hsdrop;
            CLASSES = c (3);
            CLUSTER = schcode;
            WITHIN = female mothed homeres expect droptht7 expel
            arrest hisp black;
            BETWEEN = lunch mstrat;
```

Input For A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
DEFINE:     lunch = lunch/100;
            mstrat = mstrat/1000;

ANALYSIS:   TYPE = MIXTURE TWOLEVEL;
            ALGORITHM = INTEGRATION;

OUTPUT:     SAMPSTAT STANDARDIZED TECH1 TECH8;

PLOT:       TYPE = PLOT3;
            SERIES = math7-math10 (s);
```


Input For A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
MODEL:      %WITHIN%

            %OVERALL%

            i s | math7@0 math8@1 math9@2 math10@3;

            i s ON female hisp black mothed homeres expect
            droptht7 expel arrest;

            c#1 c#2 ON female hisp black mothed homeres expect
            droptht7 expel arrest;

            hsdrop ON female hisp black mothed homeres expect
            droptht7 expel arrest;
```

Input For A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
%c#1%
[i*40 s*1];
math7-math10*20;
i*13 s*3;

%c#2%
[i*40 s*5];
math7-math10*30;
i*8 s*3;
i s ON female hisp black mothed homeres expect
dropth7 expel arrest;

%c#3%
[i*45 s*3];
math7-math10*10;
i*34 s*2;
i s ON female hisp black mothed homeres expect
dropth7 expel arrest;
```


Input For A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
%BETWEEN%

%OVERALL%

ib | math7-math10@1; [ib@0];

ib*1; hsdrop*1; ib WITH hsdrop;
math7-math10@0;

ib ON lunch mstrat;

c#1 c#2 ON lunch mstrat;

hsdrop ON lunch mstrat;

%c#1%
[hsdrop$1*-.3];

%c#2%
[hsdrop$1*.9];

%c#3%
[hsdrop$1*1.2];
```

Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement

Summary of Data

```
Number of patterns          13
Number of y patterns        13
Number of u patterns         1
Number of clusters          44

Size (s)     Cluster ID with Size s

   12           304
   13           305
   38           112
   39           109
   40           138
   42           120
   43           307
   44           303
   45           143       146
```


Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
   46           101
   48           144       106
   51           102       308
   52           136       118     133     111
   53           140       142     108     131     122     124
   54           301       117     127     137     126
   55           103       141     123
   56           110
   57           147
   58           121       105     145     135
   59           119
   73           104
   89           302
   94           309
  118           115
```

Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
MAXIMUM LOG-LIKELIHOOD VALUE FOR THE UNRESTRICTED (H1) MODEL IS
-36393.088

THE STANDARD ERRORS OF THE MODEL PARAMETER ESTIMATES MAY NOT BE
TRUSTWORTHY FOR SOME PARAMETERS DUE TO A NON-POSITIVE DEFINITE
FIRST-ORDER DERIVATIVE PRODUCT MATRIX. THIS MAY BE DUE TO THE
STARTING VALUES BUT MAY ALSO BE AN INDICATION OF MODEL
NONIDENTIFICATION. THE CONDITION NUMBER IS -0.758D-16. PROBLEM
INVOLVING PARAMETER 54.

THE NONIDENTIFICATION IS MOST LIKELY DUE TO HAVING MORE
PARAMETERS THAN THE NUMBER OF CLUSTERS. REDUCE THE NUMBER OF
PARAMETERS.

THE MODEL ESTIMATION TERMINATED NORMALLY
```


Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

Tests Of Model Fit

```
Loglikelihood
             H0 Value                        -26247.205
Information Criteria
             Number of Free Parameters              122
             Akaike (AIC)                     52738.409
             Bayesian (BIC)                   53441.082
             Sample-Size Adjusted BIC         53053.464
                (n* = (n + 2) / 24)
             Entropy                              0.632

FINAL  CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE BASED
ON ESTIMATED POSTERIOR PROBABILITIES

 Class 1        686.43905          0.29285
 Class 2        430.83877          0.18380
 Class 3       1226.72218          0.52335
```

Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Between Level					
CLASS 1					
IB ON					
LUNCH	-1.805	1.310	-1.378	-0.851	-0.176
MSTRAT	-13.365	3.086	-4.331	-6.299	-0.448
HSDROP ON					
LUNCH	1.087	0.543	2.004	1.087	0.290
MSTRAT	-0.178	1.478	-0.120	-0.178	-0.016
IB WITH					
HSDROP	-0.416	0.328	-1.267	-0.196	-0.253

Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Intercepts					
MATH7	0.000	0.000	0.000	0.000	0.000
MATH8	0.000	0.000	0.000	0.000	0.000
MATH9	0.000	0.000	0.000	0.000	0.000
MATH10	0.000	0.000	0.000	0.000	0.000
IB	0.000	0.000	0.000	0.000	0.000
Residual Variances					
HSDROP	0.550	0.216	2.542	0.550	0.915
MATH7	0.000	0.000	0.000	0.000	0.000
MATH8	0.000	0.000	0.000	0.000	0.000
MATH9	0.000	0.000	0.000	0.000	0.000
MATH10	0.000	0.000	0.000	0.000	0.000
IB	3.456	1.010	3.422	0.768	0.768

Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

	Estimates	S.E.	Est./S.E.
Within Level			
C#1 ON			
FEMALE	-0.751	0.188	-3.998
HISP	0.094	0.705	0.133
BLACK	0.900	0.385	2.339
MOTHED	-0.003	0.106	-0.028
HOMERES	-0.060	0.069	0.864
EXPECT	-0.251	0.074	-3.406
DROPTHT7	1.616	0.451	3.583
EXPEL	0.698	0.337	2.068
ARREST	1.093	0.384	2.842

Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

	Estimates	S.E.	Est./S.E.
C#2 ON			
FEMALE	-1.610	0.450	-3.577
HISP	1.144	0.466	2.458
BLACK	-0.961	0.656	-1.465
MOTHED	-0.234	0.139	-1.684
HOMERES	0.102	0.094	1.085
EXPECT	0.056	0.089	0.628
DROPTHT7	0.570	0.657	0.869
EXPEL	1.217	0.397	3.068
ARREST	1.133	0.580	1.951
Intercepts			
C#1	0.492	0.535	0.921
C#2	-0.533	0.627	-0.849

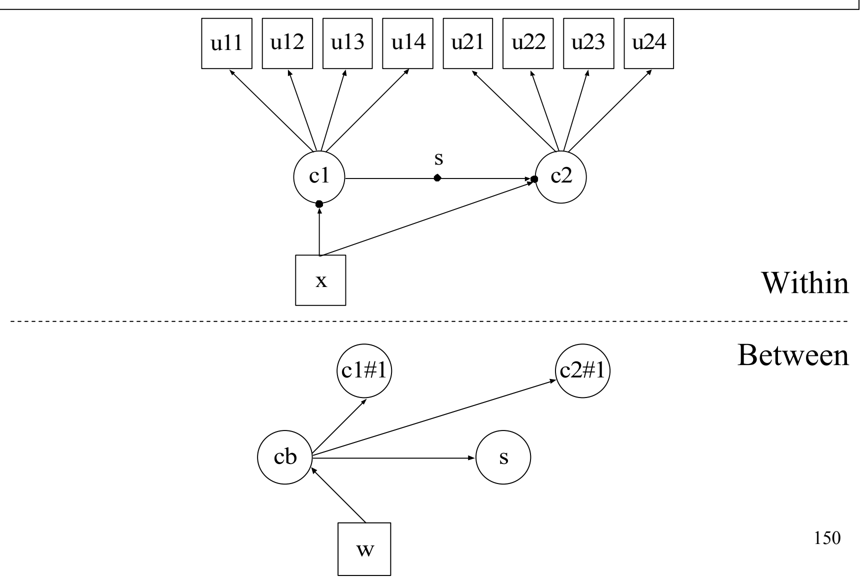
Output Excerpts A Multilevel Growth Mixture Model For LSAY Math Achievement (Continued)

```
                    Estimates    S.E.  Est./S.E.

Between Level

C#1         ON
    LUNCH             2.265     0.706     3.208
    MSTRAT           -2.876     2.909    -0.988

C#2         ON
    LUNCH            -0.088     1.343    -0.065
    MSTRAT           -0.608     2.324    -0.262
```


Between-Level Latent Classes

Latent Transition Analysis

UG Ex. 10.3 : Two-Level LTA With A Covariate And A Between-Level Categorical Latent Variable



Input For Two-Level LTA

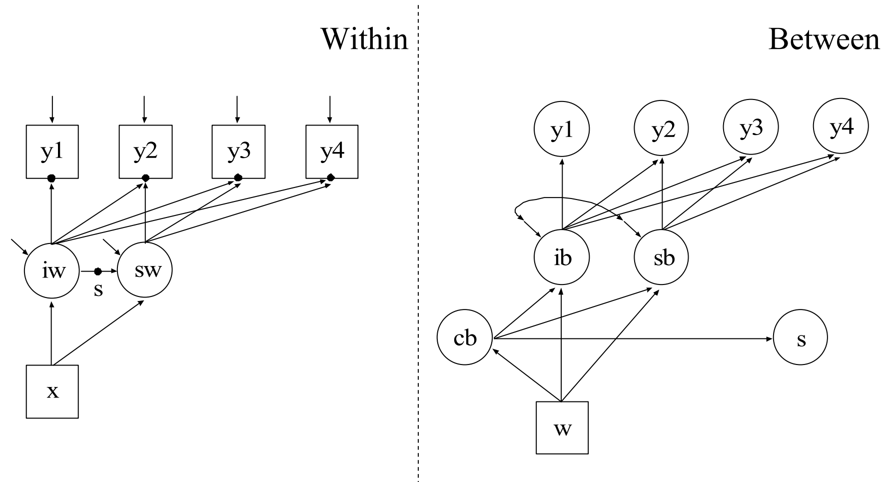
```
TITLE:      this is an example of a two-level LTA with a covariate
            and a between-level categorical latent variable
DATA:       FILE = ex8.dat;
VARIABLE:   NAMES ARE u11-u14 u21-u24 x w dumb dum1 dum2 clus;
            USEVARIABLES = u11-w;
            CATEGORICAL = u11-u14 u21-u24;
            CLASSES = cb(2) c1(2) c2(2);
            WITHIN = x;
            BETWEEN = cb w;
            CLUSTER = clus;
ANALYSIS:   TYPE = TWOLEVEL MIXTURE;
            PROCESSORS = 2;
MODEL:
            %WITHIN%
            %OVERALL%
            c2#1 ON c1#1 x;
            c1#1 ON x;
            %BETWEEN%
            %OVERALL%
            c1#1 ON cb#1;
```


Input For Two-Level LTA (Continued)

```
            c2#1 ON cb#1;
            cb#1 ON w;
MODEL cb:
            %WITHIN%
            %cb#1%
            c2#1 ON c1#1;
MODEL c1:
            %BETWEEN%
            %c1#1%
            [u11$1-u14$1] (1-4);
            %c1#2%
            [u11$1-u14$1] (5-8);
MODEL c2:
            %BETWEEN%
            %c2#1%
            [u21$1-u24$1] (1-4);
            %c2#2%
            [u21$1-u24$1] (5-8);
OUTPUT:     TECH1 TECH8;
```

Growth Modeling

UG Ex. 10.8: Two-Level Growth Model For A Continuous Outcome (Three-Level Analysis) With A Between-Level Categorical Latent Variable



Input For Two-Level Growth Model

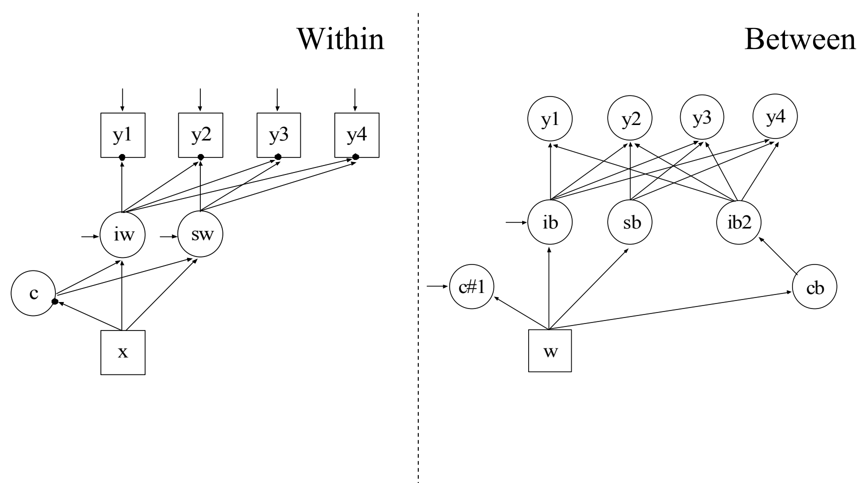
```
TITLE:      this is an example of a two-level growth model for a
            continuous outcome (three-level analysis) with a
            between-level  categorical latent variable
DATA:       FILE = ex5.dat;
VARIABLE:   NAMES ARE y1-y4 x w dummy clus;
            USEVARIABLES = y1-w;
            CLASSES = cb(2);
            WITHIN = x;
            BETWEEN = cb w;
            CLUSTER = clus;
ANALYSIS:   TYPE = TWOLEVEL MIXTURE RANDOM;
            PROCESSORS = 2;
MODEL:
            %WITHIN%
            %OVERALL%
            iw sw | y1@0 y2@1 y3@2 y4@3;
            y1-y4 (1);
            iw sw ON x;
            s | sw ON iw;
```


Input For Two-Level Growth Model (Continued)

```
            %BETWEEN%
            %OVERALL%
            ib sb | y1@0 y2@1 y3@2 y4@3;
            y1-y4@0;
            ib sb ON w;
            cb#1 ON w;
            s@0;
            %cb#1%
            [ib sb s];
            %cb#2%
            [ib sb s];
OUTPUT:     TECH1 TECH8;
```

Growth Mixture Modeling

UG Ex.10.10: Two-Level GMM (Three-Level Analysis) For A Continuous Outcome With A Between-Level Categorical Latent Variable



Input For Two-Level GMM (Three-Level Analysis)

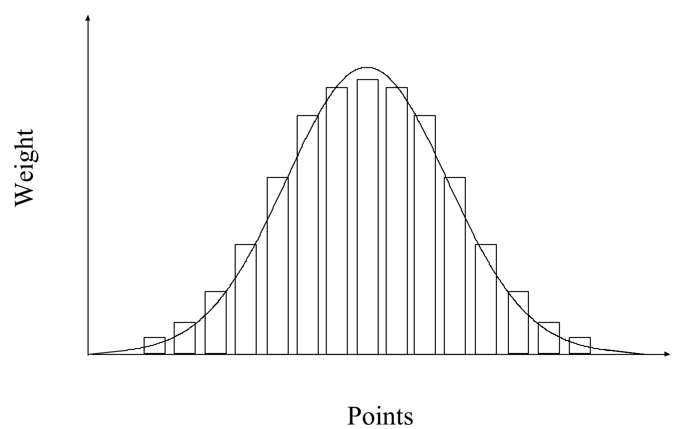
```
TITLE:      this is an example of a two-level GMM (three-level
            analysis) for a continuous outcome with a between-
            level categorical latent variable
DATA:       FILE = ex6.dat;
VARIABLE:   NAMES ARE y1-y4 x w dummyb dummy clus;
            USEVARIABLES = y1-w;
            CLASSES = cb(2) c(2);
            WITHIN = x;
            BETWEEN = cb w;
            CLUSTER = clus;
ANALYSIS:   TYPE = TWOLEVEL MIXTURE;
            PROCESSORS = 2;
MODEL:
            %WITHIN%
            %OVERALL%
            iw sw | y1@0 y2@1 y3@2 y4@3;
            iw sw ON x;
            c#1 ON x;
            %BETWEEN%
            %OVERALL%
```


Input For Two-Level GMM (Continued)

```
   ib sb | y1@0 y2@1 y3@2 y4@3;
   ib2 | y1-y4@1;
   y1-y4@0;
   ib sb ON w;
   c#1 ON w;
   sb@0; c#1;
   ib2@0;
   cb#1 ON w;
MODEL c:
   %BETWEEN%
   %c#1%
   [ib sb];
   %c#2%
   [ib sb];
MODEL cb:
   %BETWEEN%
   %cb#1%
   [ib2@0];
   %cb#2%
   [ib2];
OUTPUT: TECH1 TECH8;
```

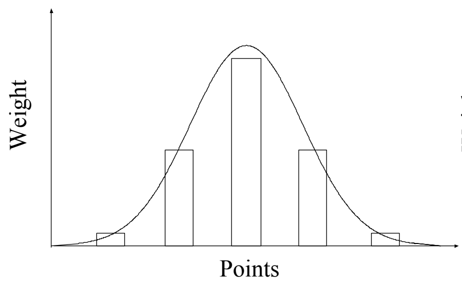
Technical Aspects Of Multilevel Modeling

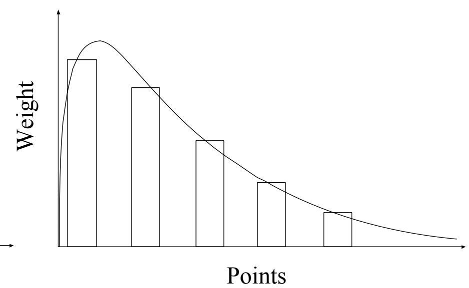
Numerical Integration
With A Normal Latent Variable Distribution



Fixed weights and points

Non-Parametric Estimation Of The Random Effect Distribution





Estimated weights and points
(class probabilities and class means)

Numerical Integration

Numerical integration is needed with maximum likelihood estimation when the posterior distribution for the latent variables does not have a closed form expression. This occurs for models with categorical outcomes that are influenced by continuous latent variables, for models with interactions involving continuous latent variables, and for certain models with random slopes such as multilevel mixture models.

When the posterior distribution does not have a closed form, it is necessary to integrate over the density of the latent variables multiplied by the conditional distribution of the outcomes given the latent variables. Numerical integration approximates this integration by using a weighted sum over a set of integration points (quadrature nodes) representing values of the latent variable.

Numerical Integration (Continued)

Numerical integration is computationally heavy and thereby time-consuming because the integration must be done at each iteration, both when computing the function value and when computing the derivative values. The computational burden increases as a function of the number of integration points, increases linearly as a function of the number of observations, and increases exponentially as a function of the dimension of integration, that is, the number of latent variables for which numerical integration is needed.

Practical Aspects Of Numerical Integration

- Types of numerical integration available in Mplus with or without adaptive quadrature
  - Standard (rectangular, trapezoid) – default with 15 integration points per dimension
  - Gauss-Hermite
  - Monte Carlo
- Computational burden for latent variables that need numerical integration
  - One or two latent variables — Light
  - Three to five latent variables — Heavy
  - Over five latent variables — Very heavy

Practical Aspects Of Numerical Integration (Continued)

- Suggestions for using numerical integration
  - Start with a model with a small number of random effects and add more one at a time
  - Start with an analysis with TECH8 and MITERATIONS=1 to obtain information from the screen printing on the dimensions of integration and the time required for one iteration and with TECH1 to check model specifications
  - With more than 3 dimensions, reduce the number of integration points to 5 or 10 or use Monte Carlo integration with the default of 500 integration points
  - If the TECH8 output shows large negative values in the column labeled ABS CHANGE, increase the number of integration points to improve the precision of the numerical integration and resolve convergence problems

Technical Aspects Of Numerical Integration

Maximum likelihood estimation using the EM algorithm computes in each iteration the posterior distribution for normally distributed latent variables f,

$$[\,f \,|\, y\,] = [\,f\,]\,[\,y \,|\, f\,] \,/\, [\,y\,], \tag{97}$$

where the marginal density for $[y]$ is expressed by integration

$$[\,y\,] = \int [\,f\,]\,[\,y \,|\, f\,]\; df. \tag{98}$$

- Numerical integration is not needed for normally distributed y - the posterior distribution is normal

Technical Aspects Of Numerical Integration (Continued)

- Numerical integration needed for:
  - Categorical outcomes u influenced by continuous latent variables f, because [u] has no closed form
  - Latent variable interactions f x x, f x y, f_1 x f_2, where [y] has no closed form, for example

$$[\,y\,] = \int [\,f_1\,, f_2\,]\,[\,y|f_1,\, f_2,\, f_1 f_2\,]\, df_1\, df_2 \qquad (99)$$

  - Random slopes, e.g. with two-level mixture modeling

Numerical integration approximates the integral by a sum

$$[\,y\,] = \int [\,f\,]\,[\,y\,|\,f\,]\, df = \sum_{k=1}^{K} w_k\,[\,y\,|\,f_k\,] \qquad (100)$$

References

Longitudinal Data

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