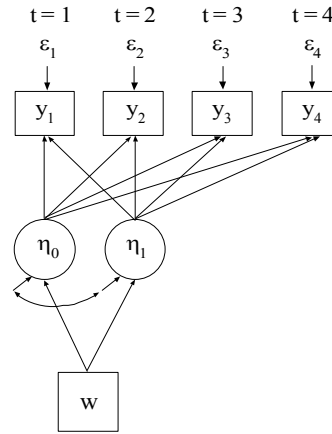
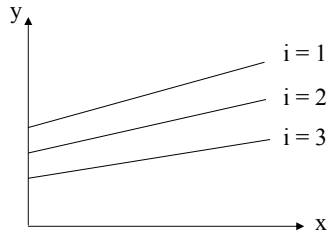


Neale & Cardon (1992)
Prescott (2004)

Multilevel Growth Models

Individual Development Over Time



(1) $y_{it} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{it}$

(2a) $\eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i}$

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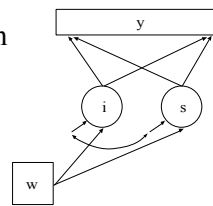
Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- Wide: Multivariate, Single-Level Approach

$$y_{it} = i_i + s_i \times \text{time}_{it} + \varepsilon_{it}$$

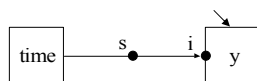
i_i regressed on w_i

s_i regressed on w_i

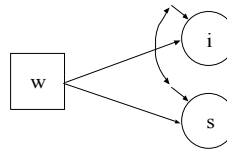


- Long: Univariate, 2-Level Approach (CLUSTER = id)

Within



Between



The intercept i is called y in Mplus

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Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long (Continued)

- Wide (one person):

	t1	t2	t3	t1	t2	t3		
Person i:	id	y1	y2	y3	x1	x2	x3	w

- Long (one cluster):

Person i:	t1	id	y1	x1	w
	t2	id	y2	x2	w
	t3	id	y3	x3	w

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Three-Level Modeling In Multilevel Terms

Time point t , individual i , cluster j .

y_{ij} : individual-level, outcome variable
 a_{1ij} : individual-level, time-related variable (age, grade)
 a_{2ij} : individual-level, time-varying covariate
 x_{ij} : individual-level, time-invariant covariate
 w_j : cluster-level covariate

Three-level analysis (Mplus considers Within and Between)

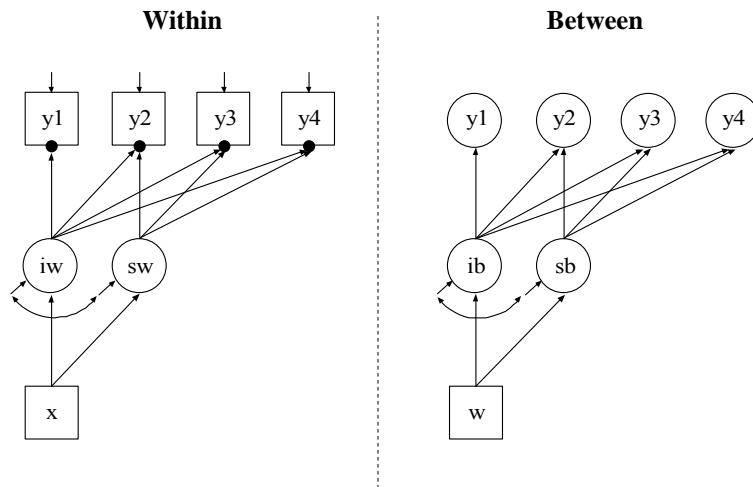
$$\text{Level 1 (Within)} : y_{ij} = \pi_{0ij} + \pi_{1ij} a_{1ij} + \pi_{2ij} a_{2ij} + e_{ij}, \quad (1)$$

$$\text{Level 2 (Within)} : \begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j} x_{ij} + r_{0ij} \rightarrow iw \\ \pi_{1ij} = \beta_{10j} + \beta_{11j} x_{ij} + r_{1ij}, \\ \pi_{2ij} = \beta_{20j} + \beta_{21j} x_{ij} + r_{2ij}. \end{cases} \quad (2)$$

$$\text{Level 3 (Between)} : \begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001} w_j + u_{00j} \rightarrow ib \\ \beta_{10j} = \gamma_{100} + \gamma_{101} w_j + u_{10j}, \\ \beta_{20j} = \gamma_{200} + \gamma_{201} w_j + u_{20j}, \\ \beta_{01j} = \gamma_{010} + \gamma_{011} w_j + u_{01j}, \\ \beta_{11j} = \gamma_{110} + \gamma_{111} w_j + u_{11j}, \\ \beta_{21j} = \gamma_{210} + \gamma_{211} w_j + u_{21j}. \end{cases} \quad (3)$$

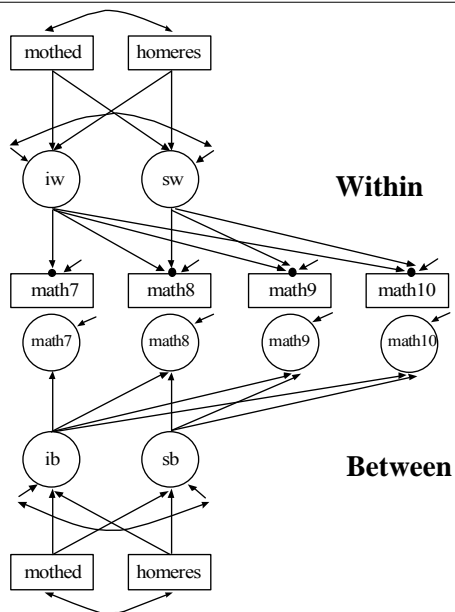
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Two-Level Growth Modeling (Three-Level Modeling)



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LSAY Two-Level Growth Model



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Input For LSAY Two-Level Growth Model With Free Time Scores And Covariates

```
TITLE:      LSAY two-level growth model with free time scores
            and covariates

DATA:      FILE IS lsay98.dat;
            FORMAT IS 3f8 f8.4 8f8.2 3f8 2f8.2;

VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
            math10 att7 att8 att9 att10 gender mothed homeres;
            USEOBS = (gender EQ 1 AND cohort EQ 2);
            MISSING = ALL (999);
            USEVAR = math7-math10 mothed homeres;
            CLUSTER = school;

ANALYSIS:  TYPE = TWOLEVEL;
            ESTIMATOR = MUML;
```

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Input For LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

```
MODEL:     %WITHIN%
            iw sw | math7@0 math8@1
            math9*2 (1)
            math10*3 (2);
            iw sw ON mothed homeres;

            %BETWEEN%
            ib sb | math7@0 math8@1
            math9*2 (1)
            math10*3 (2);
            ib sb ON mothed homeres;

OUTPUT     SAMPSTAT STANDARDIZED RESIDUAL;
```

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Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates

Summary of Data

Number of clusters 50

Size (s) Cluster ID with Size s

1	114	
2	136	
6	132	304

34	104
39	309
40	302

Average cluster size 18.627

Estimated Intraclass Correlations for the Y Variables

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
MATH7	0.199	MATH8	0.149	MATH9	0.168
MATH10	0.165				161

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Tests Of Model Fit

Chi-square Test of Model Fit

Value	24.058*
Degrees of Freedom	14
P-Value	0.0451

CFI / TLI

CFI	0.997
TLI	0.995

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.028
----------	-------

SRMR (Standardized Root Mean Square Residual)

Value for Between	0.048
Value for Within	0.007

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**Output Excerpts LSAY Two-Level Growth Model
With Free Time Scores And Covariates (Continued)**

Model Results

Within Level

SW	BY					
	MATH8	1.000	0.000	0.000	1.073	0.128
	MATH9	2.487	0.163	15.220	2.670	0.288
	MATH10	3.589	0.223	16.076	3.853	0.368
IW	ON					
	MOTHEd	1.780	0.232	7.665	0.246	0.226
	HOMERES	0.892	0.221	4.031	0.124	0.173
SW	ON					
	MOTHEd	0.053	0.063	0.836	0.049	0.045
	HOMERES	0.135	0.044	3.047	0.125	0.176

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**Output Excerpts LSAY Two-Level Growth Model
With Free Time Scores And Covariates (Continued)**

SW	WITH					
	IW	2.112	0.522	4.044	0.273	0.273
HOMERES	WITH					
	MOTHEd	0.261	0.039	6.709	0.261	0.203
Residual Variances						
	MATH7	12.748	1.434	8.888	12.748	0.197
	MATH8	12.298	0.893	13.771	12.298	0.174
	MATH9	14.237	1.132	12.578	14.237	0.166
	MATH10	24.829	2.230	11.133	24.829	0.226
	IW	47.060	3.069	15.333	0.903	0.903
	SW	1.110	0.286	3.879	0.964	0.964
Variances						
	MOTHEd	0.841	0.049	17.217	0.841	1.000
	HOMERES	1.970	0.069	28.643	1.970	1.000

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Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

		Estimates	S.E.	Est./S.E.	Std	StdYX
Between Level						
SB	BY					
	MATH8	1.000	0.000	0.000	0.196	0.052
	MATH9	2.487	0.163	15.220	0.488	0.119
	MATH10	3.589	0.223	16.076	0.704	0.115
IB	ON					
	MOTHEd	-1.225	2.587	-0.474	-0.362	-0.107
	HOMERES	7.160	1.847	3.876	2.117	1.011
SB	ON					
	MOTHEd	0.995	0.647	1.538	5.073	1.493
	HOMERES	0.017	0.373	0.045	0.086	0.041
SB	WITH					
	IB	0.382	0.248	1.538	0.575	0.575

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Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

HOMERES	WITH					
	MOTHEd	0.103	0.019	5.488	0.103	0.733
Residual Variances						
	MATH7	2.059	0.552	3.732	2.059	0.153
	MATH8	0.544	0.268	2.033	0.544	0.039
	MATH9	0.105	0.213	0.493	0.105	0.006
	MATH10	1.395	0.504	2.767	1.395	0.067
	IB	1.428	1.690	0.845	0.125	0.125
	SB	-0.051	0.071	-0.713	-1.321	-1.321
Variances						
	MOTHEd	0.087	0.023	3.801	0.087	1.000
	HOMERES	0.228	0.056	4.066	0.228	1.000
Means						
	MOTHEd	2.307	0.043	53.277	2.307	7.838
	HOMERES	3.108	0.062	50.375	3.108	6.509
Intercepts						
	IB	33.510	2.678	12.512	9.909	9.909
	SB	0.163	0.776	0.210	0.830	0.830

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**Output Excerpts LSAY Two-Level Growth Model
With Free Time Scores And Covariates (Continued)**

R-Square

Within Level

Observed Variable	R-Square
----------------------	----------

MATH7	0.803
MATH8	0.826
MATH9	0.834
MATH10	0.774

Latent Variable	R-Square
--------------------	----------

IW	0.097
SW	0.036

167

**Output Excerpts LSAY Two-Level Growth Model
With Free Time Scores And Covariates (Continued)**

R-Square

Between Level

Observed Variable	R-Square
----------------------	----------

MATH7	0.847
MATH8	0.961
MATH9	0.994
MATH10	0.933

Latent Variable	R-Square
--------------------	----------

IW	0.875
SW	Undefined 0.23207E+01

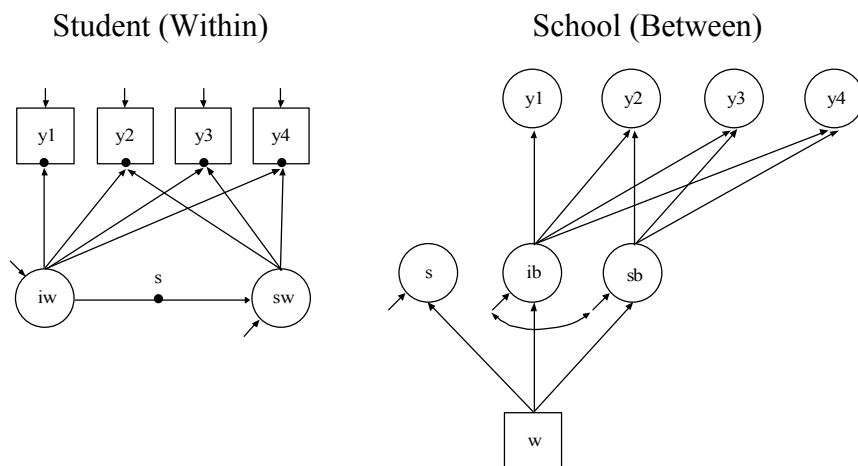
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Further Readings On Three-Level Growth Modeling

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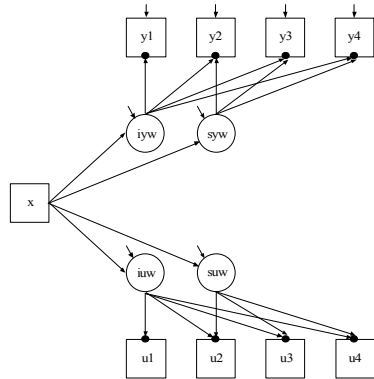
Multilevel Modeling With A Random Slope For Latent Variables



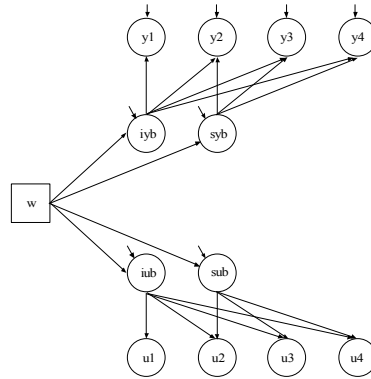
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Two-Level, Two-Part Growth Modeling

Within



Between

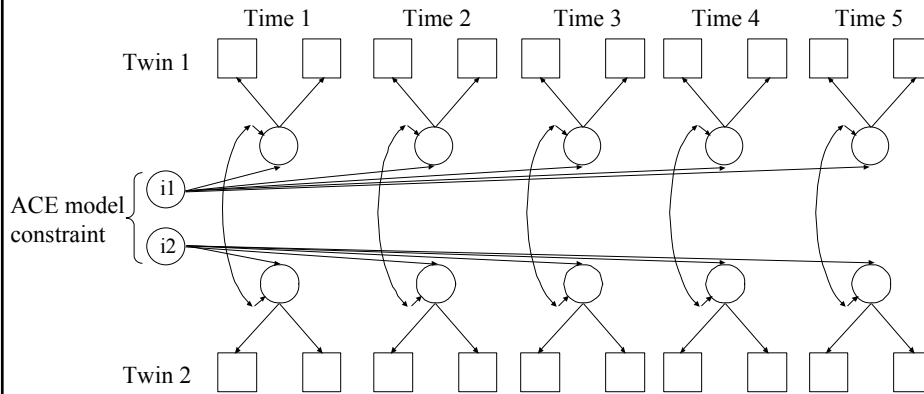


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Multiple Indicator Growth Modeling As Two-Level Analysis

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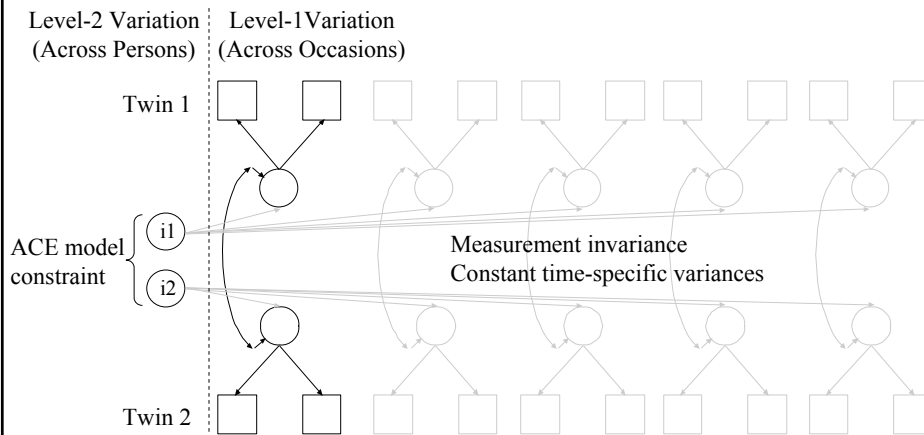
Wide Data Format, Single-Level Approach



20 variables, 12 factors, 10 dimensions of integration for ML
ML very hard, WLS easy

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Long Format, Two-Level Approach



4 variables, 2 Level-2 and 2 Level-1 factors, 4 dimensions of integration for ML
ML feasible, WLS in development

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Multilevel Discrete-Time Survival Analysis

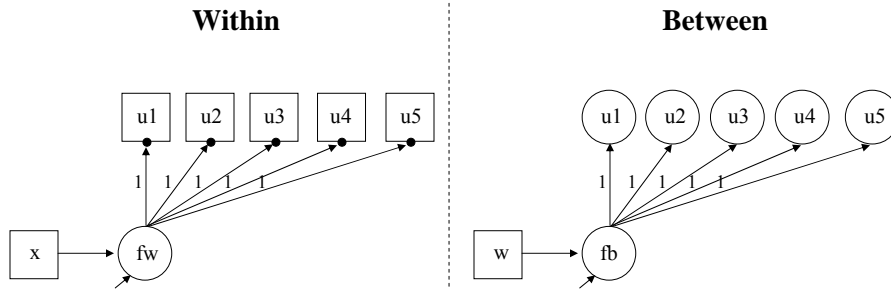
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Multilevel Discrete-Time Survival Analysis

- Muthén and Masyn (2005) in Journal of Educational and Behavioral Statistics
- Masyn dissertation
- Asparouhov and Muthén

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Multilevel Discrete-Time Survival Frailty Modeling



Vermunt (2003)

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(To request a Muthén paper, please email bmuthen@ucla.edu.)

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Mplus Analysis

Asparouhov, T. & Muthén, B. (2003a). Full-information maximum-likelihood estimation of general two-level latent variable models. In preparation.

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