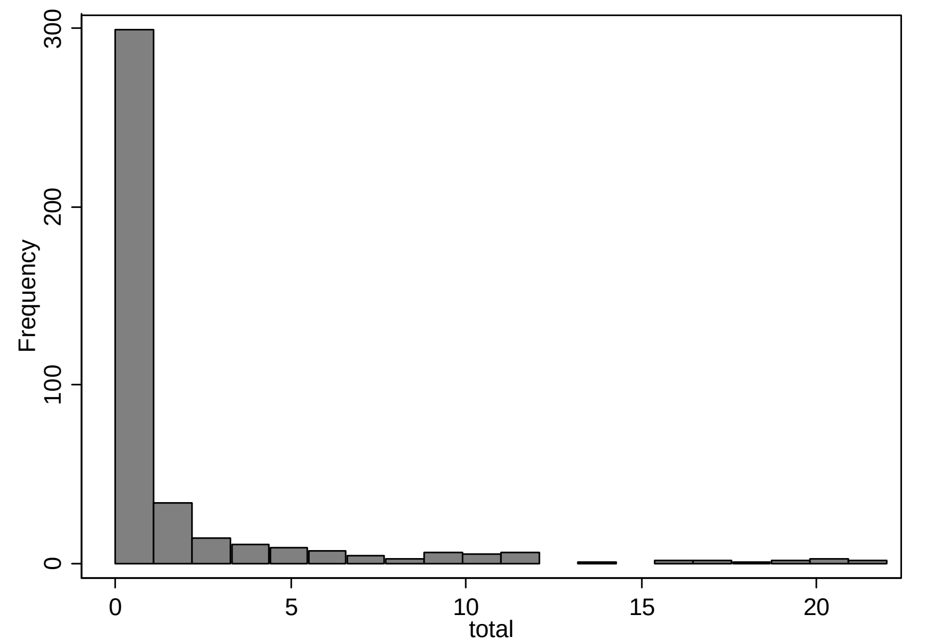
# Longitudinal Models with Zero Inflation

Frauke Kreuter, UCLA Statistics

# Data

- Characteristics
  - Large number of zeros
  - Skewed

- Examples
  - Convictions
  - Drinking
  - Drug abuse



# “Classic” examples

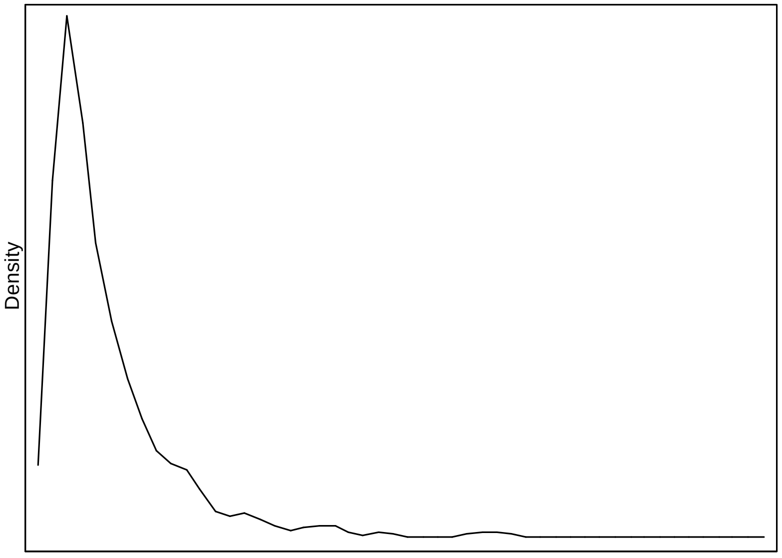
- Manufacturing applications
- Economics
- Medicine
- Public health
- Environmental science
- Education

# Example – Criminal behavior

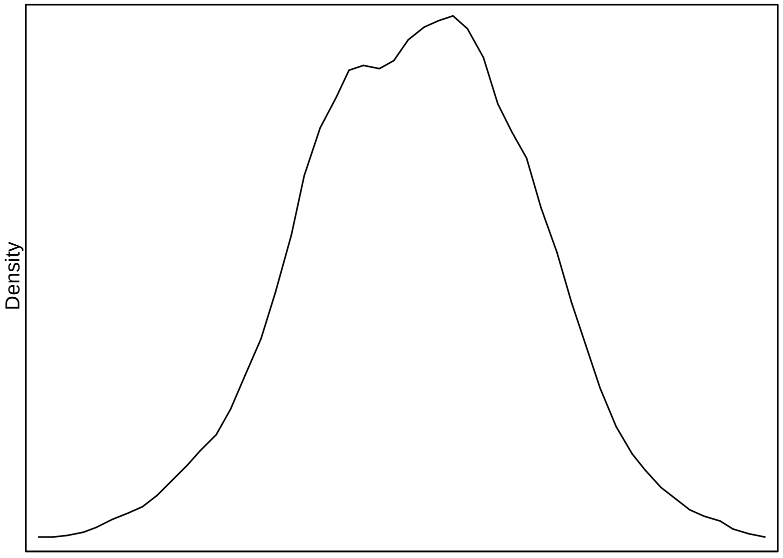
- “Cambridge study” (Farrington/West 1990)
- N = 411 (403)
- Age 10 to 40
- Number of convictions each year
- 60 percent never convicted
- In any given year 98.5% to 88.8% zero
- Biannual 97.1% to 83.2% zero

# Distributions

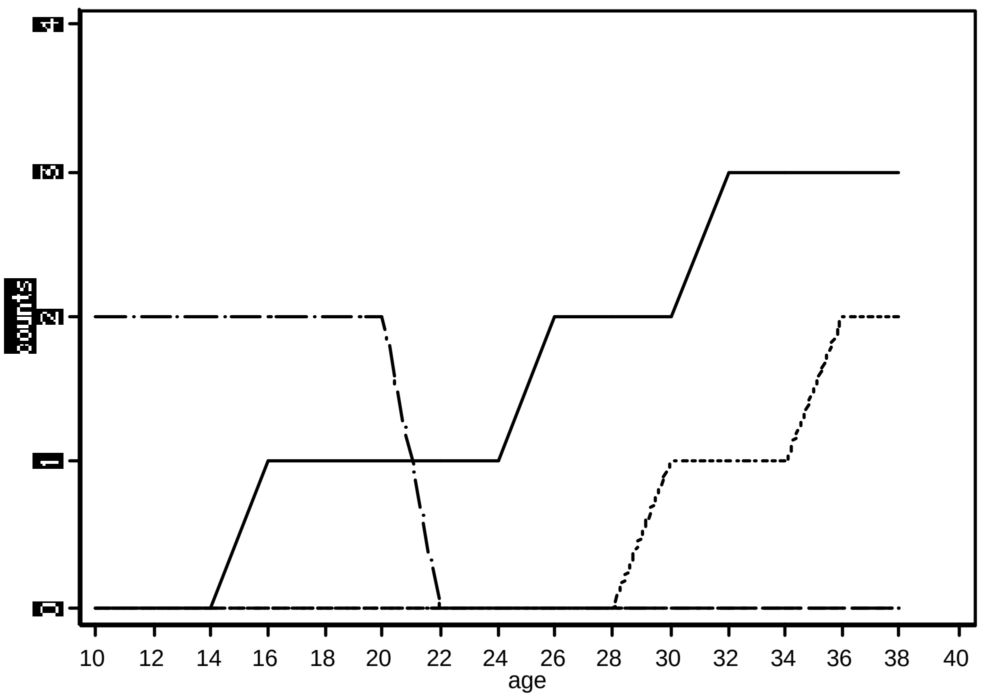
- Skewed




- Log transformed

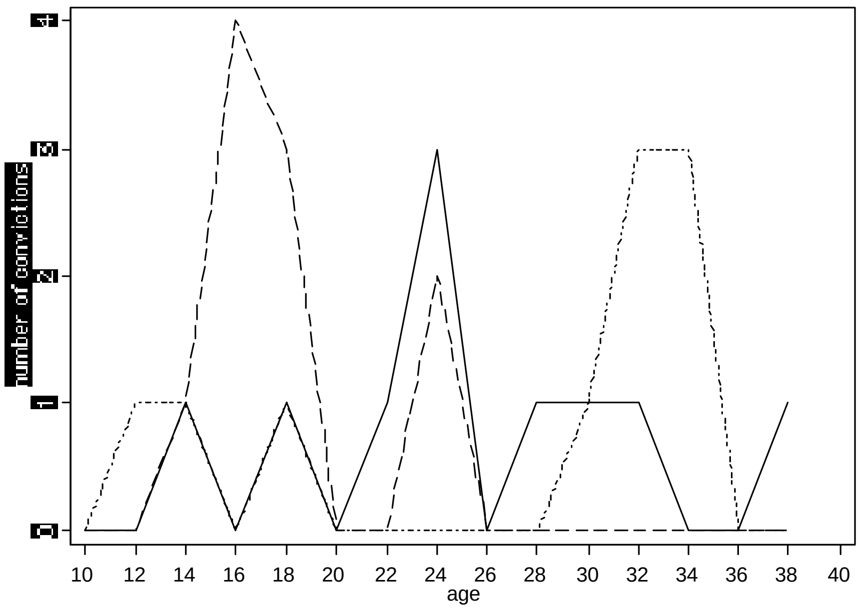


# Zeros in longitudinal data




- Onset
  - Zero until onset
  - Once behavior shown we want to model counts
- “Offset”
  - Zero after a certain point
  - E.g. abstinence, “jail time”

# "Types" of zeros




- ``Careers''
  - In and out of zero (random zeros)
  - Measurement errors
  - Zero throughout (partly structural zeros)

# One/Two-class models: Modeling alternatives

- One-class models
  - Censored normal
  - *Two-part modeling (Olsen & Schafer 2001)*
  - Onset followed by growth (Albert & Shih 2003)
- Two-class models
  - *Inflated models*
    - Zero inflated poisson
    - Censored inflated
  - Two-part modeling (mixture in 0-1 part)
  - *Two-class  (Carlin et al. 2001)*
- Multi-class models

# Modeling alternatives: Multi-class models

- Mixture censored
- Two-part models with mixture in 0-1 and growth part
  - LTA mover-stayer model
  - Two-class model for 0-1 part
- Onset followed by growth with mixture
- *LCGA with zero inflation (Roeder et al. 1999)*
- *GMM with zero class (Mplus V3)*

# Substantive questions reg. zeros

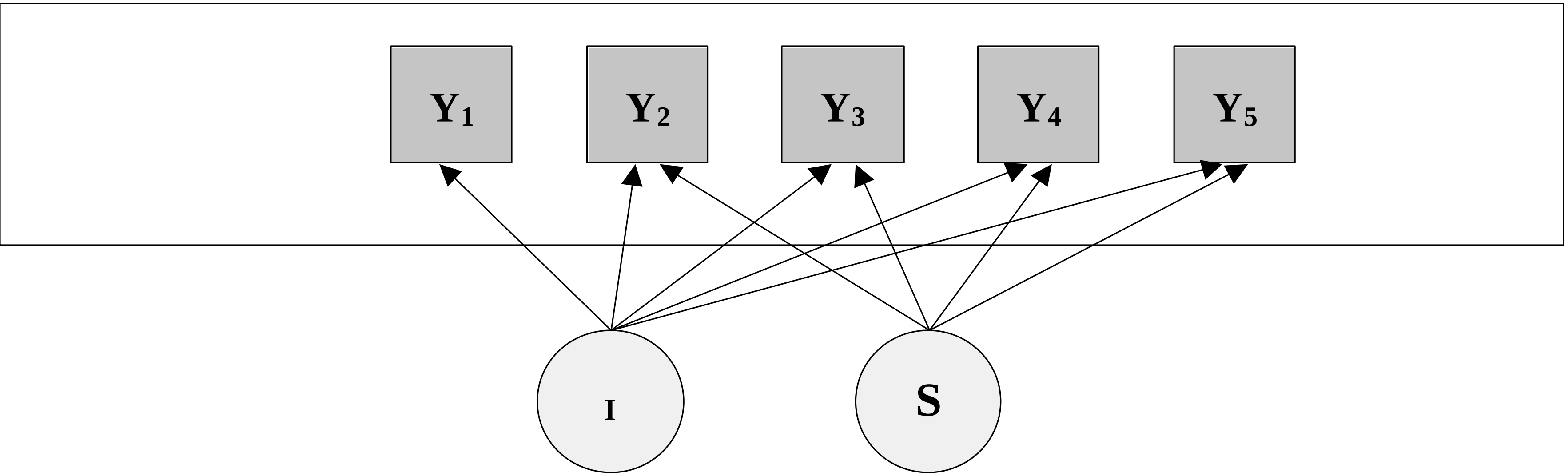
- What is the process that generates zeros?
- Do we assume a mixture of zeros?
- Who should contribute to trajectory classes?

- Are covariates allowed to have different effects on zeros and positive outcomes?
- Are different covariates allowed for zeros and positive outcomes?

# Separate process for zeros

# Example – Olsen & Schafer (2001)

- Data n=1961
  - Panel of Adolescent Prevention Trial
  - Middle school and high school students
  - Grade 7 trough 11
- Variables
  - Self reported recent alcohol use
  - Parental monitoring, rebelliousness, gender




$$y_{it} = I_{0i} + S_{1i} a_{it} + e_{it}$$

$$I_{0i} = a_{0} + V_{0i}$$

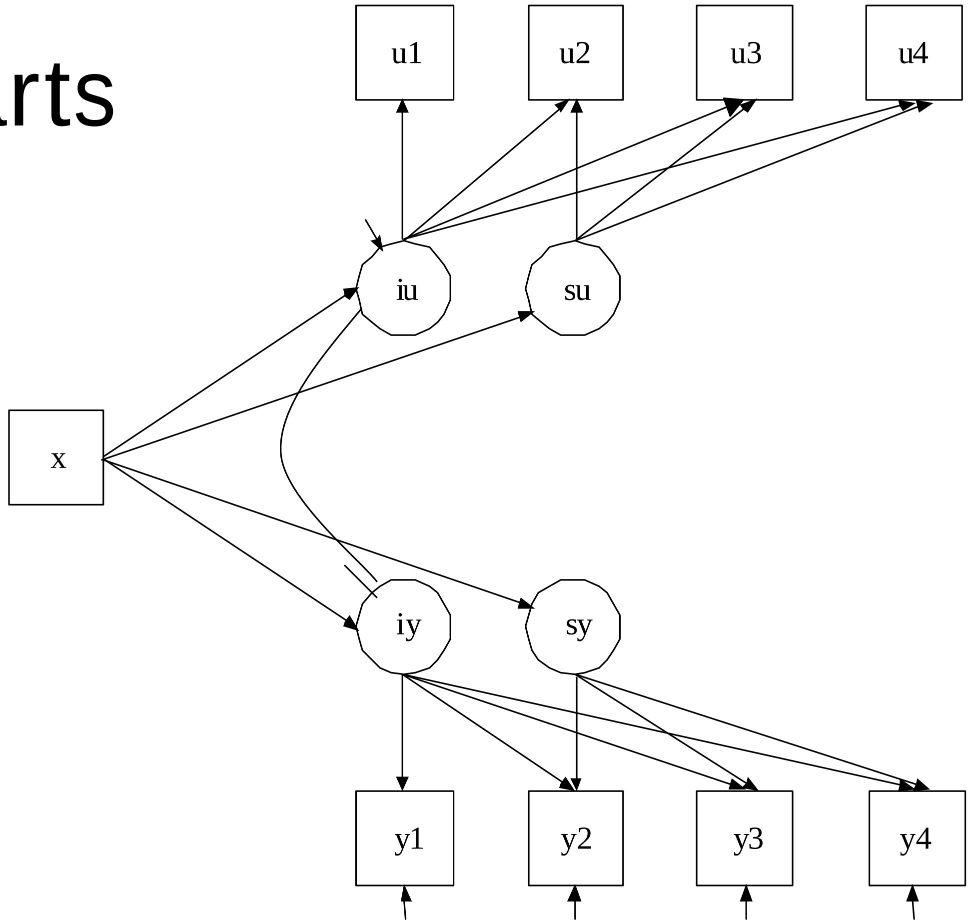
$$S_{1i} = a_{1} + V_{1i}$$

Assume, for simplicity ,

$$a_{it} = a_{t} = 0, t_1, t_2, ..., T$$

# Example – Olsen & Schafer (2001)

- Model: two-parts
  - U-part
    - Logit
    - use, no-use
  - Y-part
    - Log-normal
    - y>0



# Two-part model

$$u_{it} = \begin{cases} 1 & if & y_{it} & > & 0 \\ 0 & if & y_{it} & = & 0 \end{cases}$$

$$y_{it} = \begin{cases} m_{it} & if & y_{it} & > & 0 \\ . & if & y_{it} & = & 0 \end{cases}$$

$$m_{it} = h_{0i} + h_{1i} a_{it} + e_{it}$$

# Raw data

| $i$ | *Grade 7* | *Grade 8* | *Grade 9* | *Grade 10* | *Grade 11* |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1.7 | 2.3 | 3 |
| 3 | 0 | 1 | 0 | 1 | 1.7 |

# U-part

| $i$ | *Grade 7* | *Grade 8* | *Grade 9* | *Grade 10* | *Grade 11* |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 1 | 0 | 1 | 1 |

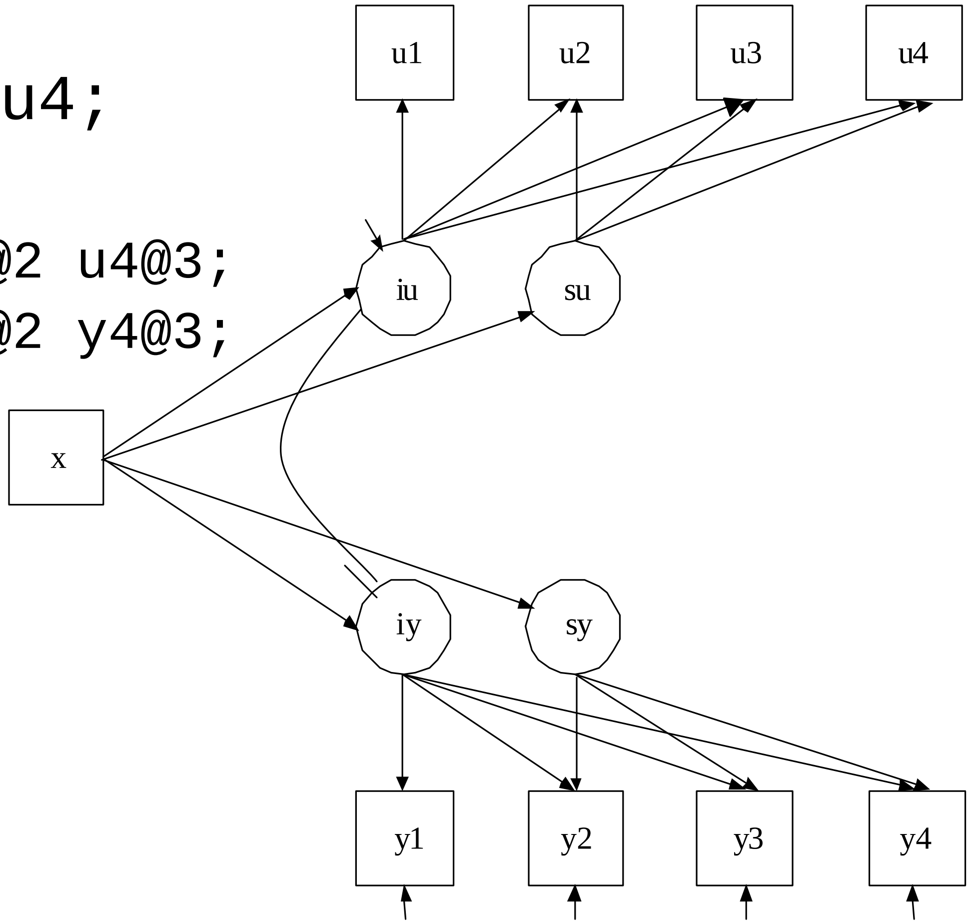
# Y-part

| *i* | *Grade 7* | *Grade 8* | *Grade 9* | *Grade 10* | *Grade 11* |
|---|---|---|---|---|---|
| 1 | . | . | . | . | . |
| 2 | . | . | 1.7 | 2.3 | 3 |
| 3 | . | 1 | . | 1 | 1.7 |

# Two part modeling - M*plus*

```
VARIABLE:…
 CATEGORICAL = u1-u4;
MODEL:
 iu su | u1@0 u2@1 u3@2 u4@3;
 iy sy | y1@0 y2@1 y3@2 y4@3;

 iu-sy on x;
 su@0;   sy@0;
 iu with su@0;
 iy with sy@0;
 iu with sy@0;
 su with iy-sy@0;
```



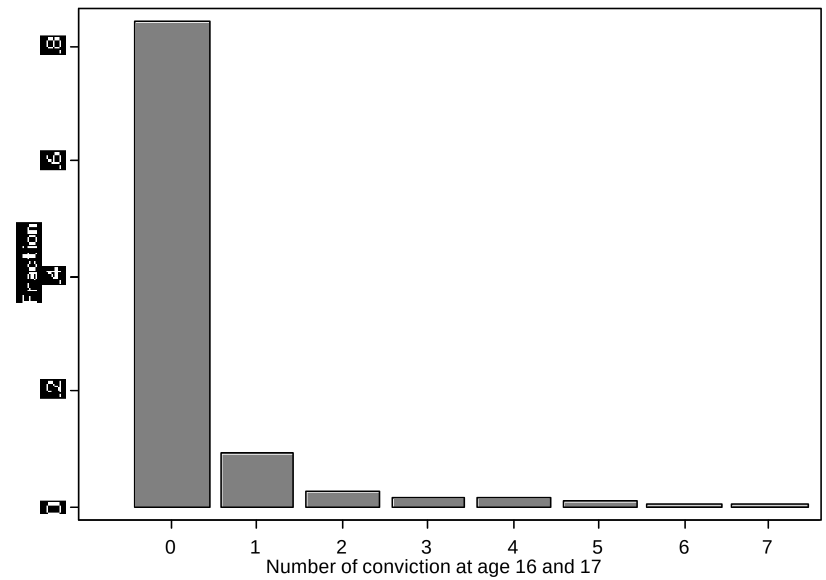
# Example – Olsen & Schafer (2001)

- U-part
  - In grade 7 unsupervised girls higher odds for drinking, effect diminishes over time
  - Low monitoring in grade 7 no effect for boys, but unsupervised boys higher odds in grade 11
- Y-part
  - Reduced monitoring increase the amount of alcohol consumption in grade 7
  - For girls effect increases over time, for boys it vanishes by grade 11

# Separate sources for zeros

# Modeling count variables (t=1)

- Models used
  - Poisson
    - With parameter $\lambda$
    - Use GLM to model $\log(\lambda)$
  - Negative binomial




- Problem
  - Zero inflation / overdispersion
  - Model assumptions don’t hold

# The poisson model

$$\Pr(Y = y) = \frac{e^{-l} l^{y}}{y!}$$ Poisson model

E(Y) = λ and V(Y) = λ

Example: Probability for count equal to 5, with lamda 4.5

$$\Pr(Y = 5) = \frac{e^{-4.5} 4.5^{5}}{5!} = .1708$$

# Zero Inflated Poisson (ZIP)

- Zero outcome can arise from one of two sources, one where outcome is always zero, another where a poisson process is at work (Lambert 1992)
- The poisson process can produce zero or another outcome
- Covariates can predict group membership, and outcome of the poisson process

# The model

$$\Pr(y_{it}=0)=\Pr(group1)+\Pr(y_{it}=0\,|\,group2)*\Pr(group2)$$

$$\Pr(y_{it}=1)=\Pr(y_{it}=1\,|\,group2)*\Pr(group2)$$

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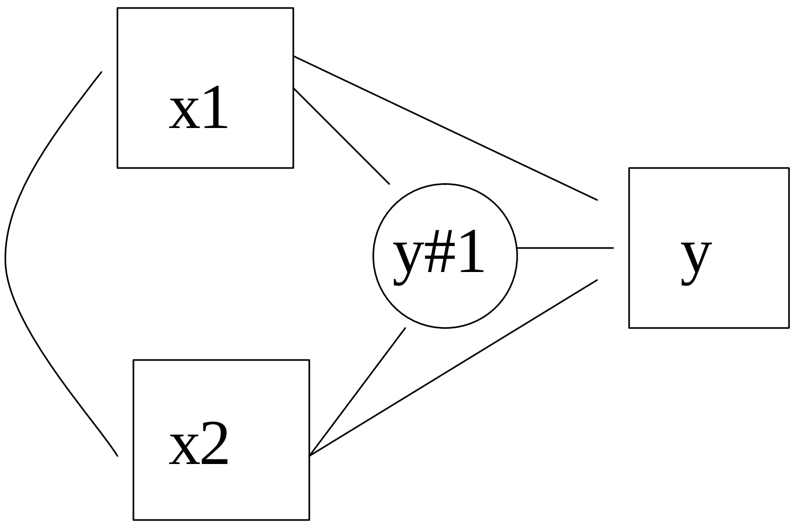
$$\Pr(group1)=\frac{e^{L}}{1+e^{L}}$$ Logistic regression model

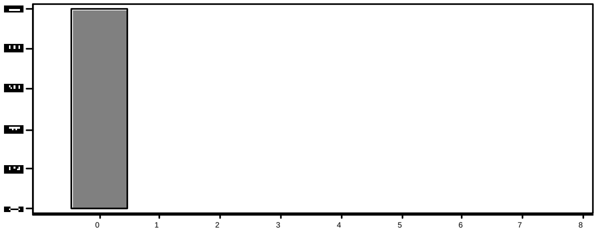
$$\Pr(\text{Y}=\text{y})=\frac{e^{-l}\,l^{y}}{y!}$$ Poisson model

# Two class models

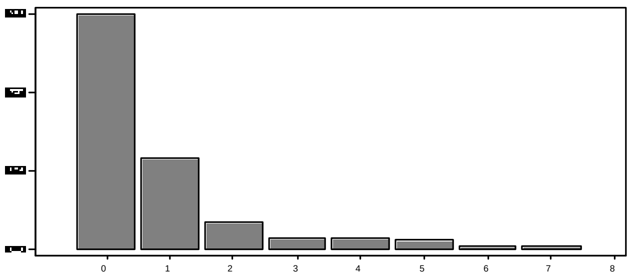



y#1 is a two class variable

y#1 = 1: “zero class”
P(y=0)=1




y#1 = 2: “convicted”
Poisson distribution



# M*plus* example - ZIP

```
!Input file

VARIABLE:
   NAMES ARE …        ;
   missing = .        ;
   USEV    = …        ;
   COUNT   = total (i);

MODEL:
   total     ON x1 x2 ;
   total#1   ON x1 x2 ;
```

```
MODEL RESULTS

                   Estimates

 TOTAL   ON
   norear          0.453
   daring          0.434

 TOTAL#1 ON
   norear         -0.260
   daring         -0.952

 Intercepts
    TOTAL#1        0.816
    TOTAL          1.031
```

ATSzip1.inp

# Interpreting the zip part

Odds of being in zero class: $e^{0.816} = 2.261$

Probability to be in zero class:

$$odds = \frac{\Pr(\text{zero\_class})}{1 - \Pr(\text{zero\_class})} = 2.26$$

$$\Pr(\text{zero\_class}) = \frac{2.26}{1 + 2.261} = .693$$

# Interpreting the count part

The average rate of conviction (average number of crimes) given a person is in the non-zero class, and both covariates are equal to zero:

$$e^{1.031} = 2.804$$

The average rate of conviction for boys with $x_1$=0 and $x_2$=0

$$= 2.804 * (1 - \Pr(\text{zero\_class}))$$

$$= 2.804 * (1 - .693)$$

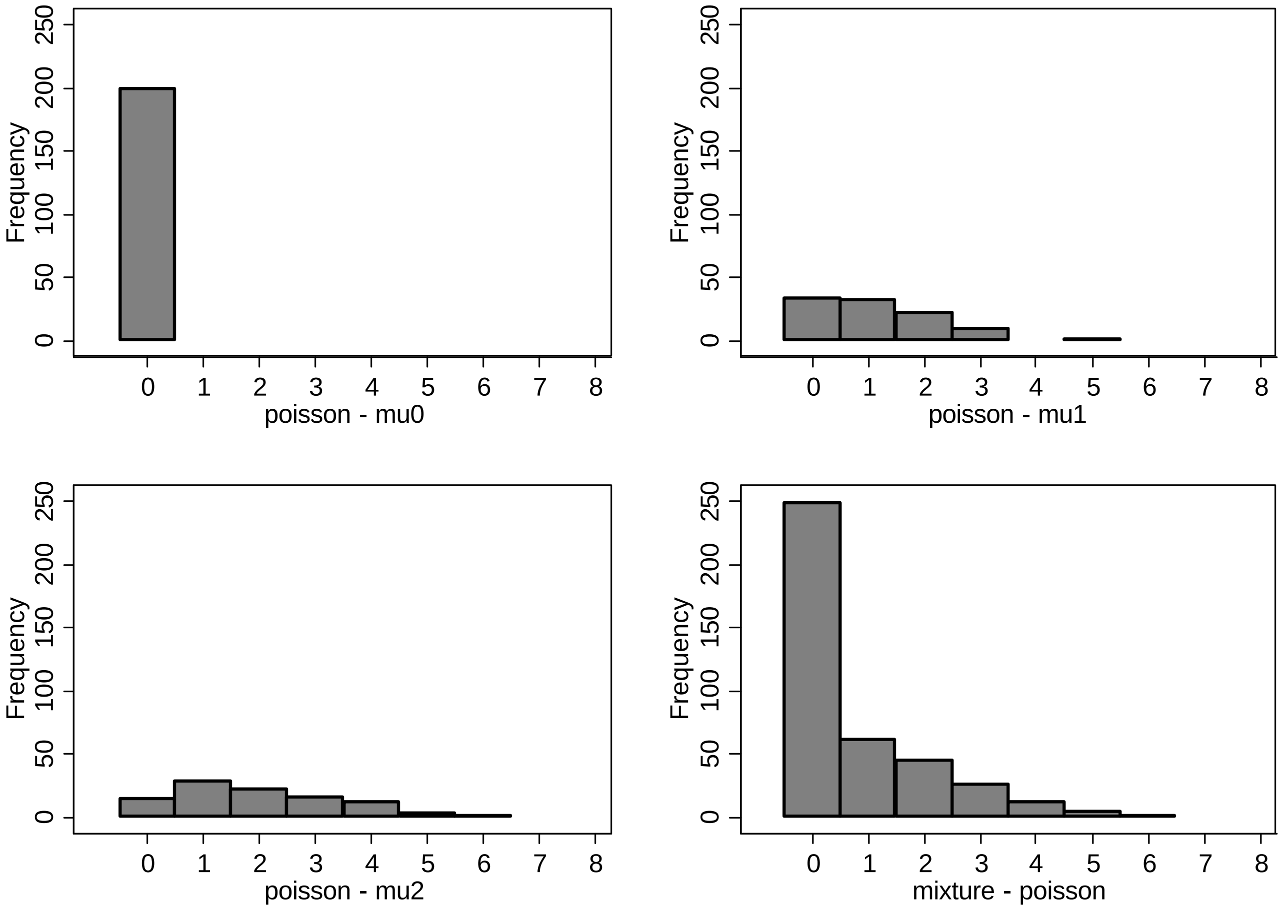
$$= .861$$

# M*plus* example – two classes

```
MODEL:
          %overall%
          total ON norear daring;
          ! class 1 has P(u=0)=1 ("zero class")
          ! class 2 is regular Poisson
          c#1 on norear daring;
          %c#1%
          [total@-15];
          total on norear@0 daring@0;
          %c#2%
```

ATSex1.inp

# Multi-class model



# Growth models

# Raw data

| $i$ | $Age\ 10$ | $Age\ 12$ | $Age\ 14$ | $Age\ 16$ | $Age\ 18$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 | 2 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

# M*plus* for (zero inflated) count

```
VARIABLE:
NAMES = …
COUNT = cage10 - cage30(i);

MODEL:
  i s  q    | cage10@0   … cage30@10;
  ii si qi  | cage10#1@0 … cage30#1@10;
   s-qi@0;
```

# M*plus* for (zero inflated) count

| | | |
|---|---|---|
| H0 Value | -1519.762 | |
| Free Param. | 4 | |
| Bayesian | 3063.530 | |
| | est. | t |
| I | -4.265 | -18.967 |
| S | 0.488 | 7.781 |
| Q | -0.056 | -8.023 |

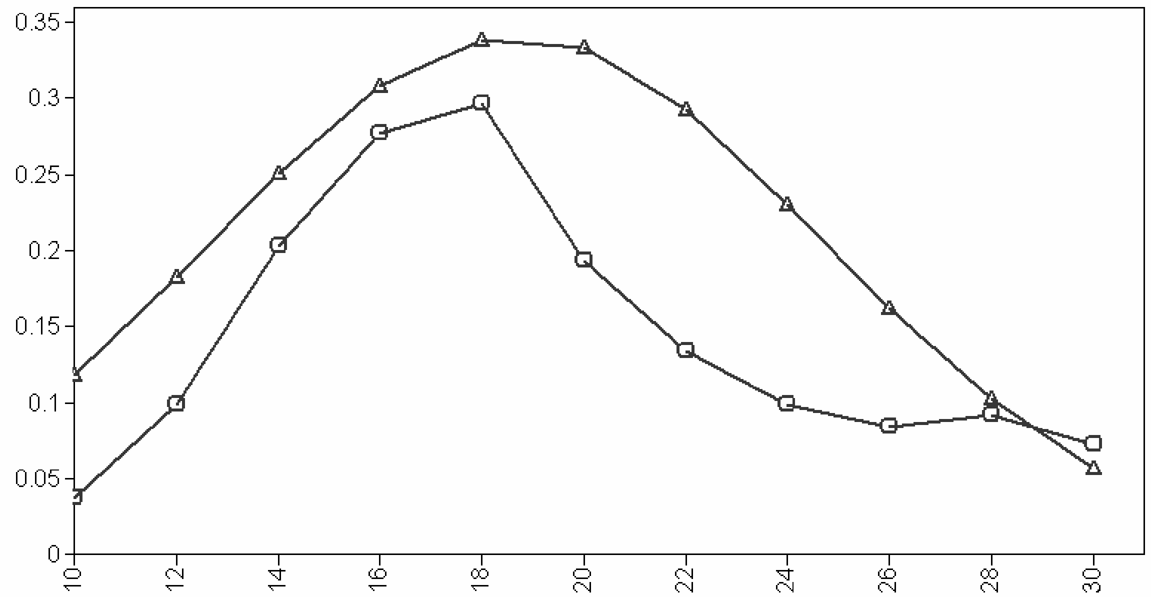
ATSgmm_1.inp

| | | |
|---|---|---|
| H0 Value | -1496.937 | |
| Free Parameter | 7 | |
| Bayesian (BIC) | 3035.878 | |
| | est. | t |
| I | -3.231 | -10.461 |
| S | 0.130 | 1.272 |
| Q | -0.027 | -2.802 |
| II | @0 | |
| SI | -1.776 | -3.765 |
| QI | 0.132 | 4.866 |
| CAGE10#1 | 1.532 | 2.861 |

ATSgmm_zip1.inp

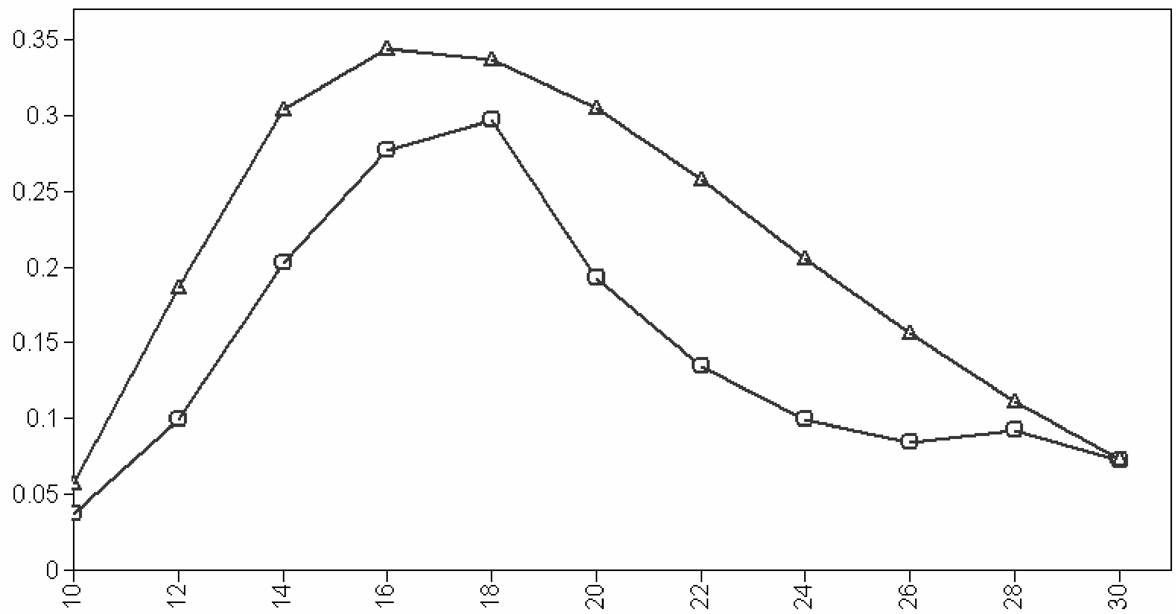
# GMM

Predicted/ observed average conviction rate




# GMM-ZIP

Predicted/ observed average convicition rate



# LCGA with ZIP
# 4-classes

```
VARIABLE:
    NAMES   = … ;
    missing = . ;
    USEV    = cage10-cage30;
    COUNT   = cage10-cage30(i);
    CLASSES = c(4);

ANALYSIS:
    TYPE      = MIXTURE;
    ALGORITHM = INTEGRATION;

MODEL:
    %OVERALL%

    i  s  q  |  cage10@0 ..   cage30@10;
    ii si qi |  cage10#1@0 .. cage30#1@10;

    i-qi@0;
```

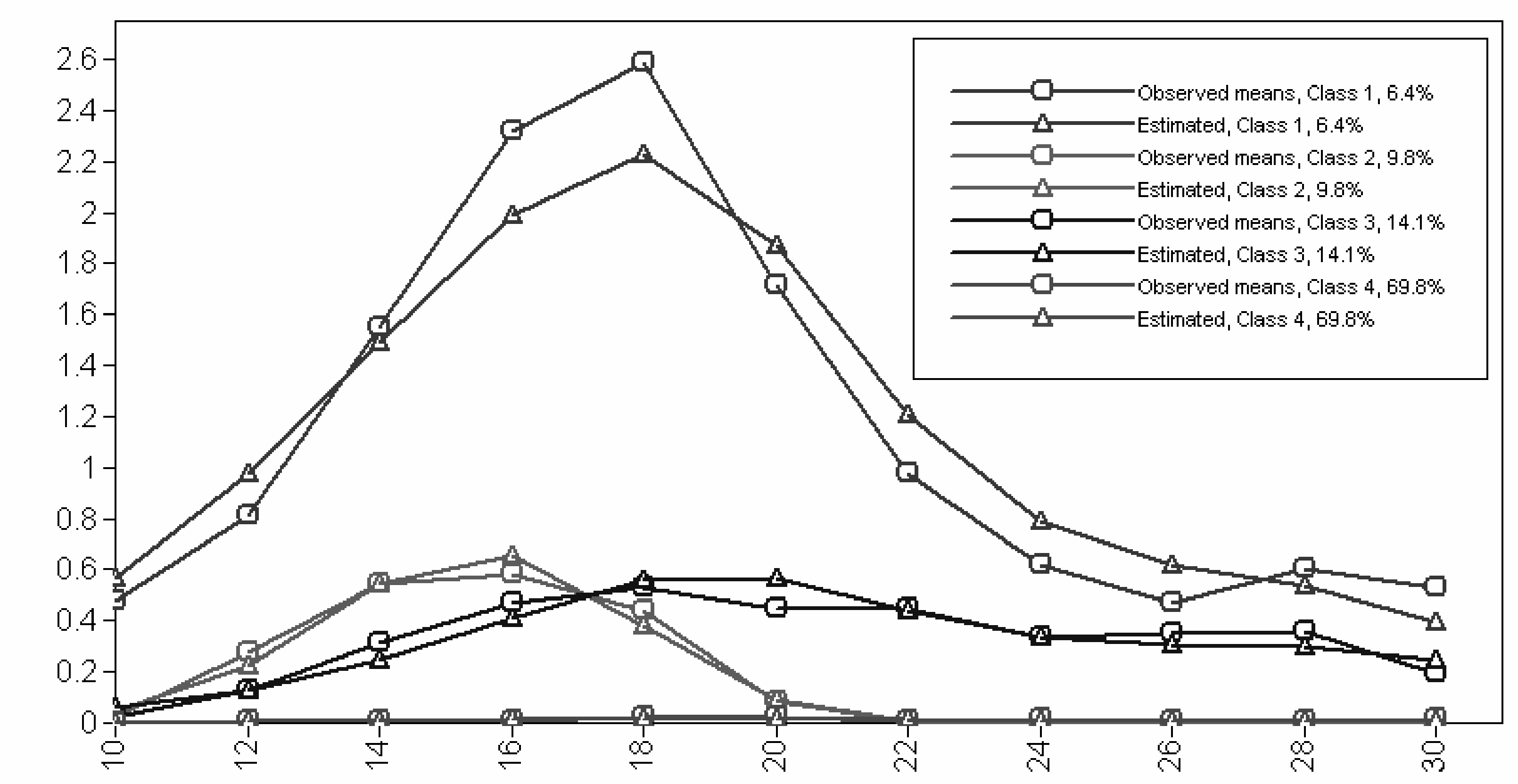
```
Loglikelihood H0 Value          -1450.004

Information Criteria
Number of Free Parameters       18
Bayesian (BIC)                  3008.033
Entropy                         0.811

LO-MENDELL-RUBIN ADJUSTED LRT TEST
         Value                  26.463
         P-Value                0.7244
```

ATS_LCGA.inp

# Outlook Latent class growth model with 4 classes



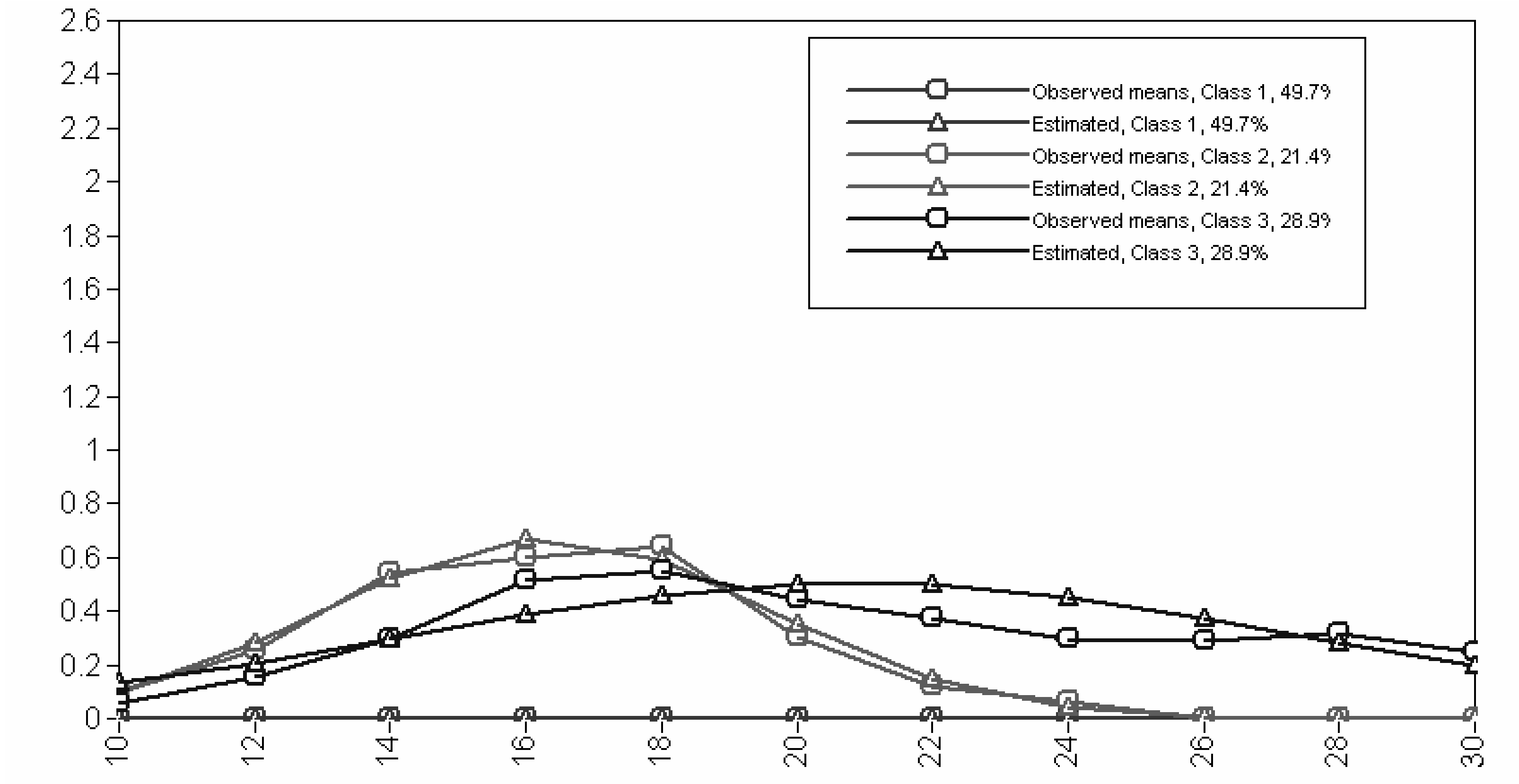
# GMM with random intercept

## using Poisson, 3 classes of which one is structural zero

```
VARIABLE:
   NAMES        = … ; missing = . ;
   USEV         = cage10-cage30;
   COUNT        = cage10-cage30;
   CLASSES      = c(3);
ANALYSIS:
   TYPE         = MIXTURE;
   ALGORITHM    = INTEGRATION;
MODEL:
   %OVERALL%
   i  s  q  |  cage10@0 ..   cage30@10;
   s-q@0;
   %c#1%  ! zero class throughout
   [i-q@0];
   [cage10-cage30@-15];
```

ATSexGMMoptseed.inp

# Results GMM zero class



# Model comparison & evaluation

- Model fit
  - Loglikelihood
  - BIC
- Predictive power for distal outcomes
- Description
  - Entropy
  - Classification table
  - Comparing classifications across models

# References (selection)


- Brown, R., Catalano, C., Fleming, C.B., Haggerty, K.P., Abbott, R.D. (2004): Adolescent Substance Use Outcomes in the Raising Healthy Children Project: A Two-Part Latent Growth Curve Analysis. Mansucript presented at SPR.
- Land, K.C., McCall, P.L., & Nagin, D.S. (1996): A Comparison of Poisson, Negative Binomial, and Semiparametric Mixed Poisson Regression Models. Sociological Methods & Research, 24, 387-442.
- Olsen, M.K. & Schafer, J.L. (2001): A Two-Part Random-Effects Model for Semicontinuous Longitudinal Data. Journal of the American Statistical Association, 96, 730-745.
- Roeder, K., Lynch, K.G. & Nagin, D.S. (1999): Modeling uncertainty in latent class membership: A case study in criminology. Journal of the American Statistical Association, 94, 766-776.