Mplus Short Courses

Topic 1

Exploratory Factor Analysis, Confirmatory Factor Analysis, And Structural Equation Modeling For Continuous Outcomes

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Table Of Contents

Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
  - V5: November 2007
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn, Jean Maninger

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Statistical Analysis With Latent Variables A General Modeling Framework

Statistical Concepts Captured By Latent Variables

Continuous Latent Variables

- Measurement errors
- Factors
- Random effects
- Frailties, liabilities
- Variance components
- Missing data

Categorical Latent Variables

- Latent classes
- Clusters
- Finite mixtures
- Missing data

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Statistical Analysis With Latent Variables A General Modeling Framework (Continued)

Models That Use Latent Variables

Continuous Latent Variables

- Factor analysis models
- Structural equation models
- Growth curve models
- Multilevel models

Categorical Latent Variables

- Latent class models
- Mixture models
- Discrete-time survival models
- Missing data models

Mplus integrates the statistical concepts captured by latent variables into a general modeling framework that includes not only all of the models listed above but also combinations and extensions of these models.

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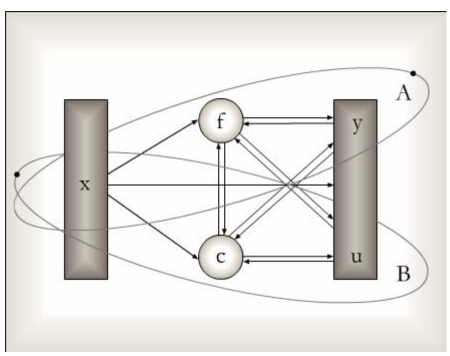
General Latent Variable Modeling Framework



- Observed variables
  - x background variables (no model structure)
  - y continuous and censored outcome variables
  - u categorical (dichotomous, ordinal, nominal) and count outcome variables
- Latent variables
  - f continuous variables
    – interactions among f's
  - c categorical variables
    – multiple c's

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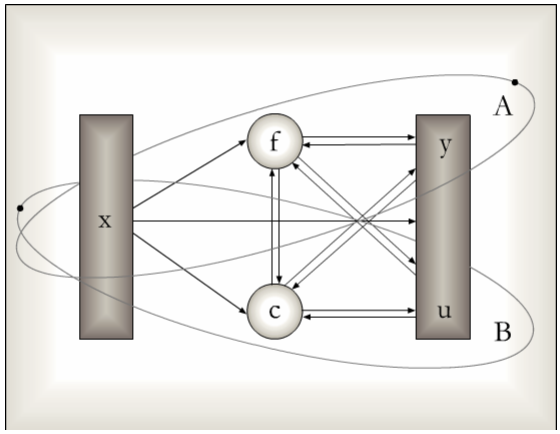
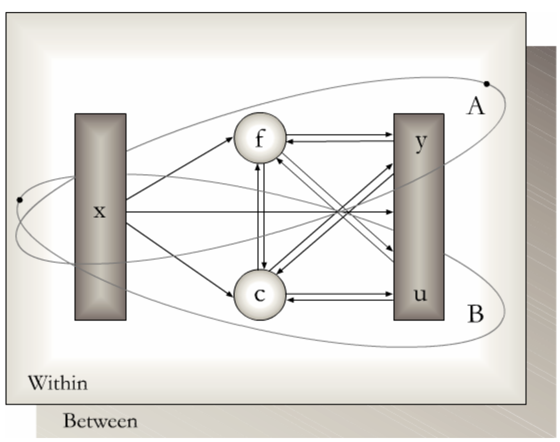
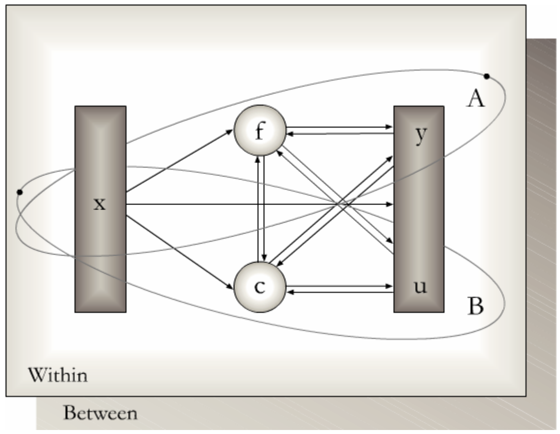
General Latent Variable Modeling Framework



General Latent Variable Modeling Framework



General Latent Variable Modeling Framework



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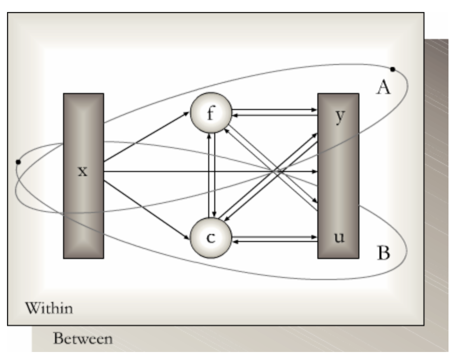
General Latent Variable Modeling Framework



- Observed variables
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Mplus

Several programs in one

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis
- Survival analysis
- Growth modeling
- Multilevel analysis
- Complex survey data analysis
- Monte Carlo simulation

Fully integrated in the general latent variable framework

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Overview

Single-Level Analysis

	Cross-Sectional	Longitudinal
Continuous Observed And Latent Variables	***Day 1*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling	***Day 2*** Growth Analysis
Adding Categorical Observed And Latent Variables	***Day 3*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling Latent Class Analysis Factor Mixture Analysis Structural Equation Mixture Modeling	***Day 4*** Latent Transition Analysis Latent Class Growth Analysis Growth Analysis Growth Mixture Modeling Discrete-Time Survival Mixture Analysis Missing Data Analysis

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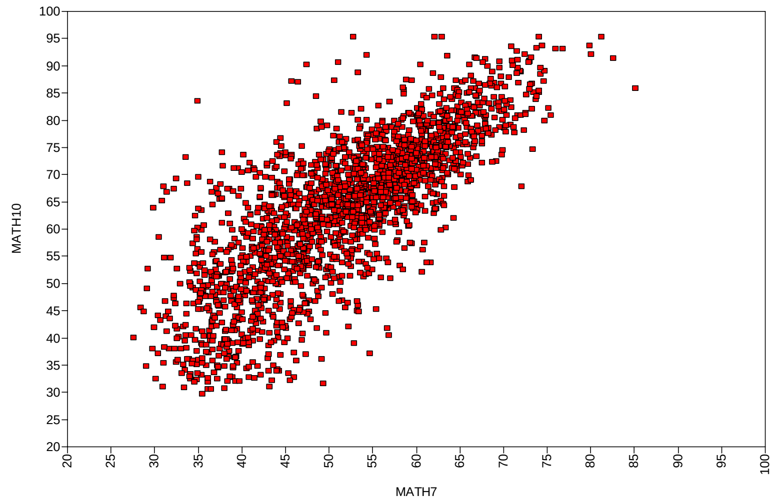
Overview (Continued)

Multilevel Analysis

	Cross-Sectional	Longitudinal
Continuous Observed And Latent Variables	***Day 5*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling	***Day 5*** Growth Analysis
Adding Categorical Observed And Latent Variables	***Day 5*** Latent Class Analysis Factor Mixture Analysis	***Day 5*** Growth Mixture Modeling

Regression Analysis

LSAY Math Regression



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Regression Analysis

Regression model:

$$y_i = \alpha + \beta\, x_i + \varepsilon_i\,, \qquad (1)$$

$$E(\varepsilon_i \mid x_i) = E(\varepsilon_i) = E(\varepsilon) = 0 \text{ (}x\text{ and }\varepsilon\text{ uncorrelated)}, \qquad (2)$$

$$V(\varepsilon_i \mid x_i) = V(\varepsilon_i) = V(\varepsilon) \text{ (constant variance)}. \qquad (3)$$

For inference and ML estimation, we also assume ε normal.

The model implies

$$E(y \mid x) = \alpha + \beta\, x \qquad \text{(conditional expectation function)}$$

$$V(y \mid x) = V(\varepsilon) \qquad \text{(homoscedasticity)}$$

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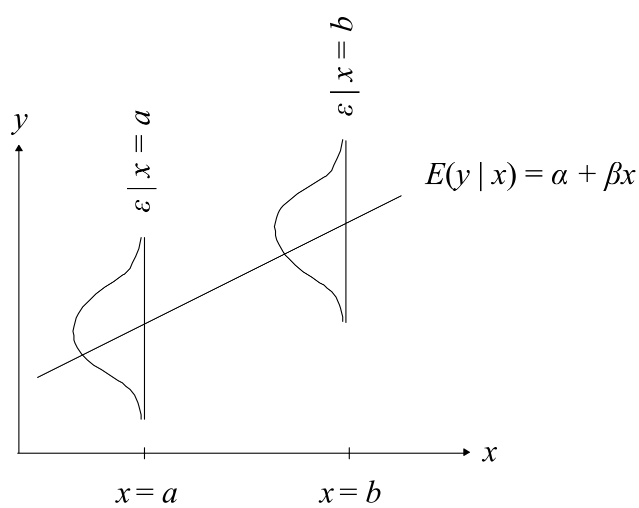
Regression Analysis (Continued)



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Regression Analysis (Continued)

Population formulas:

$$y_i = \alpha + \beta\, x_i + \varepsilon_i\,, \qquad (1)$$

$$E(y) = E(\alpha) + E(\beta\, x) + E(\varepsilon)$$

$$= \alpha + \beta\, E(x) \qquad (2)$$

$$V(y) = V(\alpha) + V(\beta\, x) + V(\varepsilon)$$

$$= \beta^2\, V(x) + V(\varepsilon) \qquad (3)$$

$$Cov(y, x) = E[y - E(y)]\,[x - E(x)] = \beta\, V(x) \qquad (4)$$

$$R^2 = \beta^2\, V(x) \,/\, (\beta^2\, V(x) + V(\varepsilon)) \qquad (5)$$

$$Stdyx\ \beta = \beta\, \frac{SD(x)}{SD(y)} \qquad (6)$$

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Regression Analysis (Continued)

The model has 3 parameters: α, β, and $V(\varepsilon)$
Note: $E(x)$ and $V(x)$ are not model parameters

Formulas for ML and OLS parameter estimates based on a random sample

$$\hat{\beta} = s_{yx} / s_{xx}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\, \bar{x}$$

$$\hat{V}(\varepsilon) = s_{yy} - \hat{\beta}^2 s_{xx}$$

Prediction

$$\hat{y}_i = \hat{\alpha} + \hat{\beta}\, x_i$$

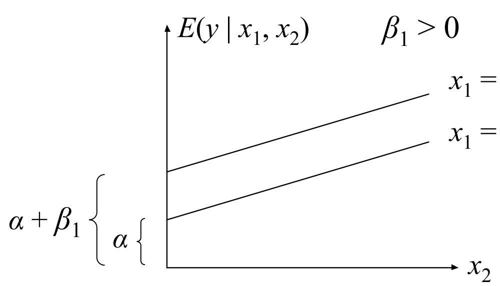
Regression Analysis (Continued)

x_1 0/1 dummy variable (e.g. gender), x_2 continuous variable

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$$

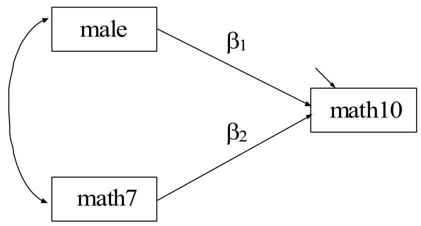
$$E(y \mid x_1 = 0, x_2) = \alpha + \beta_2 x_2$$

$$E(y \mid x_1 = 1, x_2) = \underbrace{\alpha + \beta_1}_{\text{intercept}} + \beta_2 x_2$$



Analogous to ANCOVA

Regression Of LSAY Math10 On Gender And Math7



Parameter estimates are produced for the intercept, the two slopes, and the residual variance.

Note: Variances and covariance for male and math7 are not part of the model

Input For Regression Of Math10 On Gender And Math7

```
TITLE:      Regressing math10 on math7 and gender
DATA:       FILE = dropout.dat;
            FORMAT = 11f8 6f8.2 1f8 2f8.2 10f2;
VARIABLE:   NAMES ARE id school gender mothed fathed fathsei ethnic
            expect pacpush pmpush homeres math7 math8 math9 math10
            math11 math12 problem esteem mathatt clocatn dlocatn
            elocatn flocatn glocatn hlocatn ilocatn jlocatn
            klocatn llocatn;
            MISSING = mothed (8) fathed (8)  fathsei (996 998)
                 ethnic (8) homeres (98) math7-math12 (996 998);
            USEVAR = math7 math10 male;
DEFINE:     male = gender - 1; ! male is a 0/1 variable created from
                          ! gender = 1/2 where 2 is male
MODEL:      math10 ON male math7;
OUTPUT:     TECH1 SAMPSTAT STANDARDIZED;
PLOT:       TYPE = PLOT1;
```

Output Excerpts For Regression Of Math10 On Gender And Math7

Estimated Sample Statistics

Means

	MATH10	MATH7	MALE
1	62.423	50.378	0.522

Covariances

	MATH10	MATH7	MALE
MATH10	186.926		
MATH7	109.826	103.950	
MALE	-0.163	-0.334	0.250

Correlations

	MATH10	MATH7	MALE
MATH10	1.000		
MATH7	0.788	1.000	
MALE	-0.024	-0.066	1.000

Output Excerpts For Regression Of Math10 On Gender And Math7 (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
MATH10 ON					
MALE	0.763	0.374	2.037	0.763	0.028
MATH7	1.059	0.018	57.524	1.059	0.790
Intercepts					
MATH10	8.675	0.994	8.726	8.675	0.635
Residual Variances					
MATH10	70.747	2.225	31.801	70.747	0.378

R-SQUARE

Observed Variable	R-Square
MATH10	0.622

Further Readings On Regression Analysis

Agresti, A. & Finlay, B. (1997). Statistical methods for the social sciences. Third edition. New Jersey: Prentice Hall.

Amemiya, T. (1985). Advanced econometrics. Cambridge, Mass.: Harvard University Press.

Hamilton, L.C. (1992). Regression with graphics. Belmont, CA: Wadsworth.

Johnston, J. (1984). Econometric methods. Third edition. New York: McGraw-Hill.

Lewis-Beck, M. S. (1980). Applied regression: An introduction. Newbury Park, CA: Sage Publications.

Moore, D.S. & McCabe, G.P. (1999). Introduction to the practice of statistics. Third edition. New York: W.H. Freeman and Company.

Pedhazur, E.J. (1997). Multiple regression in behavioral research. Third Edition. New York: Harcourt Brace College Publishers.

Path Analysis

Path Analysis

Used to study relationships among a set of observed variables

- Estimate and test direct and indirect effects in a system of regression equations
- Estimate and test theories about the absence of relationships

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Maternal Health Project (MHP) Data

The data are taken from the Maternal Health Project (MHP). The subjects were a sample of mothers who drank at least three drinks a week during their first trimester plus a random sample of mothers who used alcohol less often.

Mothers were measured at the fourth and seventh month of pregnancy, at delivery, and at 8, 18, and 36 months postpartum. Offspring were measured at 0, 8, 18 and 36 months.

Variables for the mothers included: demographic, lifestyle, current environment, medical history, maternal psychological status, alcohol use, tobacco use, marijuana use, and other illicit drug use. Variables for the offspring included: head circumference, height, weight, gestational age, gender, and ethnicity.

Data for the analysis include mother's alcohol and cigarette use in the third trimester and the child's gender, ethnicity, and head circumference both at birth and at 36 months.

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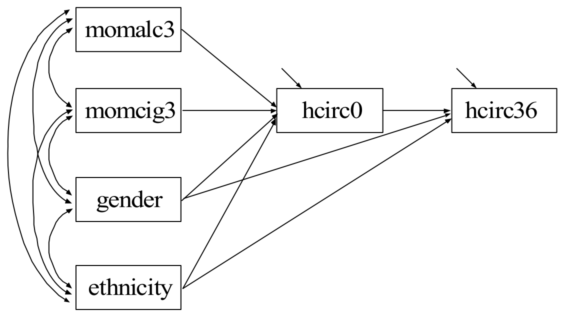

Input For Maternal Health Project Path Analysis

```
TITLE:      Maternal health project path analysis

DATA:       FILE IS headalln.dat;
            FORMAT IS 1f8.2 47f7.2;

VARIABLE:   NAMES ARE id weight0 weight8 weight18 weigh36
            height0 height8 height18 height36 hcirc0 hcirc8
            hcirc18 hcirc36 momalc1 momalc2 momalc3 momalc8
            momalc18 momalc36 momcig1 momcig2 momcig3 momcig8
            momcig18 momcig36 gender eth momht gestage age8
            age18 age36 esteem8 esteem18 esteem36 faminc0
            faminc8 faminc18 faminc36 momdrg36 gravid sick8
            sick18 sick36 advp advm1 advm2 advm3;

            MISSING = ALL (999);

            USEV = momalc3 momcig3 hcirc0 hcirc36 gender eth;

            USEOBS = id NE 1121 AND NOT (momalc1 EQ 999 AND
                     momalc2 EQ 999 AND momalc3 EQ 999);
```

Input For Maternal Health Project Path Analysis (Continued)

```
DEFINE:     hcirc0  = hcirc1/10;
            hcirc36 = hcirc36/10;
            momalc3 = log(momalc3 +1);

MODEL:      hcirc36 ON hcirc0 gender eth;
            hcirc0 ON momalc3 momcig3 gender eth;

OUTPUT:     SAMPSTAT STANDARDIZED;
```

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Output Excerpts Maternal Health Project Path Analysis

Tests Of Model Fit

```
Chi-Square Test of Model Fit
             Value                              1.781
             Degrees of Freedom                     2
             P-Value                            .4068

RMSEA (Root Mean Square Error Of Approximation)
             Estimate                            .000
             90 Percent C.I.                     .000  0.079
             Probability RMSEA <= .05            .774
```

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Output Excerpts Maternal Health Project Path Analysis (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
HCIRC36 ON					
HCIRC0	.415	.036	11.382	.415	.439
GENDER	.762	.107	7.146	.762	.270
ETH	-.094	.107	-.879	-.094	-.033
HCIRC0 ON					
MOMALC3	-.500	.239	-2.090	-.500	-.084
MOMCIG3	-.013	.005	-2.604	-.013	-.108
GENDER	.495	.118	4.185	.495	.166
ETH	.578	.125	4.625	.578	.194

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Output Excerpts Maternal Health Project Path Analysis (Continued)

Residual Variances					
HCIRC0	2.043	.119	17.107	2.043	.920
HCIRC36	1.385	.087	15.844	1.385	.697
Intercepts					
HCIRC0	33.729	.112	301.357	33.729	22.629
HCIRC36	35.338	1.227	28.791	35.338	25.069

R-Square

Observed Variable	R-Square
HCIRC0	.080
HCIRC36	.303

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The MODEL INDIRECT Command

MODEL INDIRECT is used to request indirect effects and their standard errors. Delta method standard errors are computed as the default.

The BOOTSTRAP option of the ANALYSIS command can be used to obtain bootstrap standard errors for the indirect effects.

The STANDARDIZED option of the OUTPUT command can be used to obtain standardized indirect effects.

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The MODEL INDIRECT Command (Continued)

The CINTERVAL option of the OUTPUT command can be used to obtain confidence intervals for the indirect effects and the standardized indirect effects. Three types of 95% and 99% confidence intervals can be obtained: symmetric, bootstrap, or bias-corrected bootstrap confidence intervals. The bootstrapped distribution of each parameter estimate is used to determine the bootstrap and bias-corrected bootstrap confidence intervals. These intervals take non-normality of the parameter estimate distribution into account. As a result, they are not necessarily symmetric around the parameter estimate.

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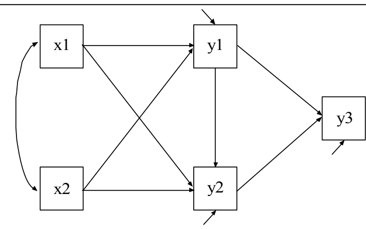
The MODEL INDIRECT Command (Continued)



MODEL INDIRECT has two options:

- IND – used to request a specific indirect effect or a set of indirect effects
- VIA – used to request a set of indirect effects that includes specific mediators

MODEL INDIRECT

```
    y3 IND y1 x1;        !x1 -> y1 -> y3
    y3 IND y2 x2;        !x2 -> y2 -> y3
    y3 IND x1;           !x1 -> y1 -> y3
                         !x1 -> y2 -> y3
                         !x1 -> y1 -> y2 -> y3
    y3 VIA y2 x1;        !x1 -> y2 -> y3
                         !x1 -> y1 -> y2 -> y3
```

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Further Readings On Path Analysis

MacKinnon, D.P., Lockwood, C.M., Hoffman, J.M., West, S.G. & Sheets, V. (2002). A comparison of methods to test mediation and other intervening variable effects. Psychological Methods, 7, 83-104.

MacKinnon, D.P., Lockwood, C.M. & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. Multivariate Behavioral Research, 39, 99-128.

Shrout, P.E. & Bolger, N. (2002). Mediation in experimental and nonexperimental studies: New procedures and recommendations. Psychological Methods, 7, 422-445.

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Measurement Errors And
Multiple Indicators Of Latent Variables

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Measurement Error

- Attenuation in correlations
- Measurement error in independent variables – attenuation in regression slopes
- Measurement error in dependent variables – increased standard errors
- Single indicator of a latent variable – known amount of measurement error can be specified
- Multiple indicators of a latent variable – measurement error can be estimated

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X With Measurement Error

Regressing on the true η

$$y_i = \alpha + \beta\,\eta_i + \varepsilon_i$$

x measures η measures with error

$$x_i = \eta_i + \delta_i$$

$V(x) = V(\eta) + V(\delta)$. Reliability($x$) = $V(\eta) / (V(\eta) + V(\delta))$

Regressing on x

$$y_i = \alpha^* + \beta^* x_i + \varepsilon_i$$

$$\beta^* = \frac{Cov(y,x)}{V(x)} = \frac{Cov(y,\eta)}{V(\eta)+V(\delta)} < \beta$$

Attenuated slope

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X With Measurement Error (Continued)

An example:

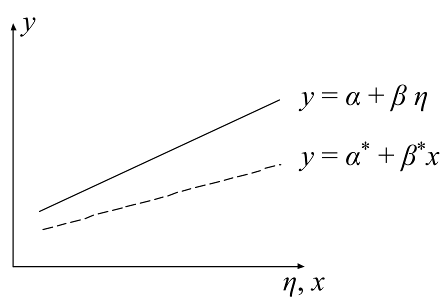
$\beta = 0.8$

$V(x) = V(\eta) + V(\delta)$

$= 1 + 0.43$

Reliability (x) = 1/(1 + 0.43) = 0.7

$\rightarrow \beta^* = 0.56$



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Measurement Error In A Single Indicator

$x_i = \nu + \lambda\, \eta_i + \varepsilon_i$

With $\lambda = 1$, $V(y) = \psi + \theta$ and reliability = $\psi / V(y)$

$V(y)$ is estimated as the sample variance, which means that reliability * sample variance = ψ and θ = (1– reliability) * sample variance.

In Mplus: f BY y@1;
y@a;

where a = θ.

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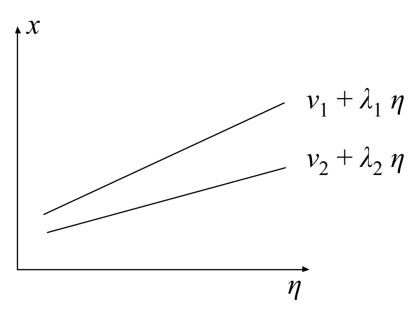
Multiple Indicators Of A Latent Variable

$$x_{1i} = \nu_1 + \lambda_1\, \eta_i + \delta_{1i}$$

$$x_{2i} = \nu_2 + \lambda_2\, \eta_i + \delta_{2i}$$



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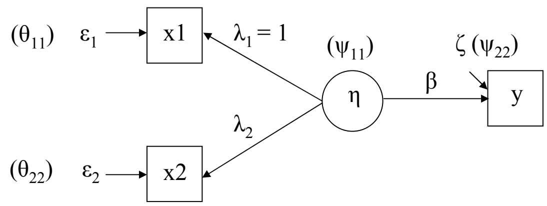
Multiple Indicators Of An Exogenous Latent Variable



Examples: Alcohol consumption during pregnancy
Dietary fat intake
Blood pressure

β gives the correct picture, free of measurement error (and the influence of collinearity)

($\beta = Cov(y_1, x_2) / Cov(x_2, x_1)$)

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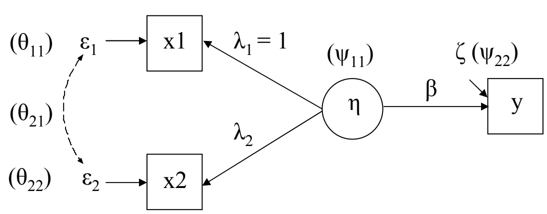
Multiple Indicators Of An Exogenous Latent Variable (Continued)



Hypothetical example 1 (β = 0.5)

Reliability(x) = 0.5
$\lambda_1 = \lambda_2 = 1$, $\psi_{11} = 0.5$, $\theta_{11} = \theta_{22} = 0.5$
$\psi_{22} = 0.75$, $R^2(y) = 0.25$
$\theta_{21} = 0.10$ (corr = 0.20)

$\beta^* = \frac{0.25}{0.5 + 0.2}$ (why? See end of day)

$= 0.36$

Hypothetical example 2 (β = 0.5)

Reliability(x) = 0.8
Change to $\psi_{11} = 0.8$

$\beta^* = \frac{0.40}{0.8 + 0.04} = 0.48$

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