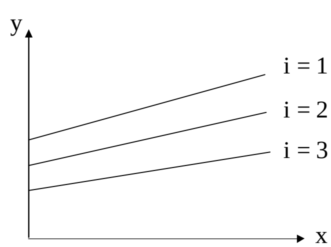
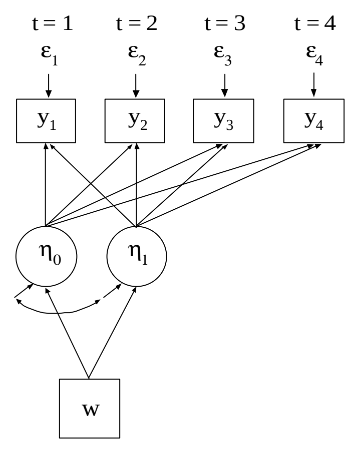
The Latent Variable Growth Model In Practice

Individual Development Over Time





(1) $y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}$

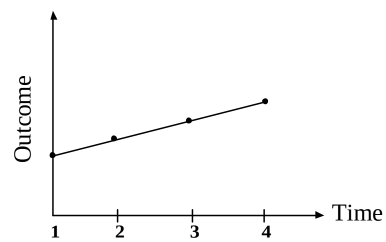
(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

Specifying Time Scores For Linear Growth Models

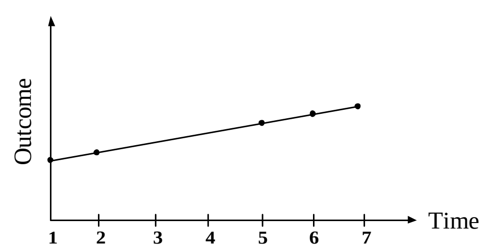
Linear Growth Model

- Need two latent variables to describe a linear growth model: Intercept and slope



- Equidistant time scores 0 1 2 3
  for slope: 0 .1 .2 .3

Specifying Time Scores For Linear Growth Models (Continued)



- Nonequidistant time scores 0 1 4 5 6
  for slope: 0 .1 .4 .5 .6

Interpretation Of The Linear Growth Factors

Model:

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \qquad (17)$$

where in the example t = 1, 2, 3, 4 and x_t = 0, 1, 2, 3:

$$y_{1i} = \eta_{0i} + \eta_{1i}\, 0 + \varepsilon_{1i}\,, \qquad (18)$$
$$\eta_{0i} = y_{1i} - \varepsilon_{1i}, \qquad (19)$$
$$y_{2i} = \eta_{0i} + \eta_{1i}\, 1 + \varepsilon_{2i}\,, \qquad (20)$$
$$y_{3i} = \eta_{0i} + \eta_{1i}\, 2 + \varepsilon_{3i}\,, \qquad (21)$$
$$y_{4i} = \eta_{0i} + \eta_{1i}\, 3 + \varepsilon_{4i}\,. \qquad (22)$$

Interpretation Of The Linear Growth Factors (Continued)

Interpretation of the intercept growth factor
η_{0i} (initial status, level):
Systematic part of the variation in the outcome variable at the time point where the time score is zero.

- Unit factor loadings

Interpretation of the slope growth factor
η_{1i} (growth rate, trend):
Systematic part of the increase in the outcome variable for a time score increase of one unit.

- Time scores determined by the growth curve shape

Interpreting Growth Model Parameters

- Intercept Growth Factor Parameters
  - Mean
    - Average of the outcome over individuals at the timepoint with the time score of zero;
    - When the first time score is zero, it is the intercept of the average growth curve, also called initial status
  - Variance
    - Variance of the outcome over individuals at the timepoint with the time score of zero, excluding the residual variance

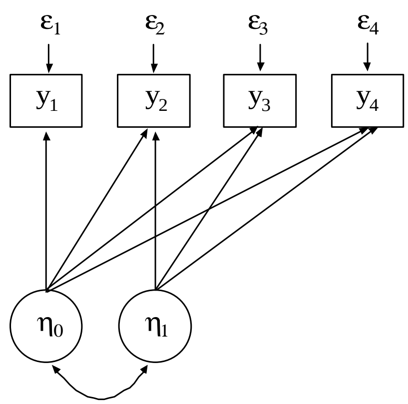
43

Interpreting Growth Model Parameters (Continued)

- Linear Slope Growth Factor Parameters
  - Mean – average growth rate over individuals
  - Variance – variance of the growth rate over individuals
  - Covariance with Intercept – relationship between individual intercept and slope values
- Outcome Parameters
  - Intercepts – not estimated in the growth model – fixed at zero to represent measurement invariance
  - Residual Variances – time-specific and measurement error variation
  - Residual Covariances – relationships between time-specific and measurement error sources of variation across time

44

Latent Growth Model Parameters And Sources Of Model Misfit



45

Latent Growth Model Parameters For Four Time Points

Linear growth over four time points, no covariates.

Free parameters in the H_1 unrestricted model:

- 4 means and 10 variances-covariances

Free parameters in the H_0 growth model:

(9 parameters, 5 d.f.):

- Means of intercept and slope growth factors
- Variances of intercept and slope growth factors
- Covariance of intercept and slope growth factors
- Residual variances for outcomes

Fixed parameters in the H_0 growth model:

- Intercepts of outcomes at zero
- Loadings for intercept growth factor at one
- Loadings for slope growth factor at time scores
- Residual covariances for outcomes at zero

46

Latent Growth Model Sources Of Misfit

Sources of misfit:
- Time scores for slope growth factor
- Residual covariances for outcomes
- Outcome variable intercepts
- Loadings for intercept growth factor

Model modifications:
- Recommended
  - Time scores for slope growth factor
  - Residual covariances for outcomes
- Not recommended
  - Outcome variable intercepts
  - Loadings for intercept growth factor

47

Latent Growth Model Parameters For Three Time Points

Linear growth over three time points, no covariates.

Free parameters in the H_1 unrestricted model:
- 3 means and 6 variances-covariances

Free parameters in the H_0 growth model
(8 parameters, 1 d.f.)
- Means of intercept and slope growth factors
- Variances of intercept and slope growth factors
- Covariance of intercept and slope growth factors
- Residual variances for outcomes

Fixed parameters in the H_0 growth model:
- Intercepts of outcomes at zero
- Loadings for intercept growth factor at one
- Loadings for slope growth factor at time scores
- Residual covariances for outcomes at zero

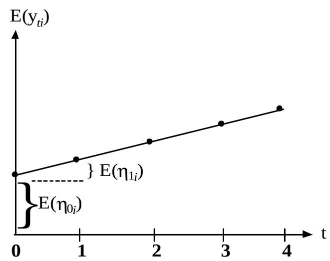
48

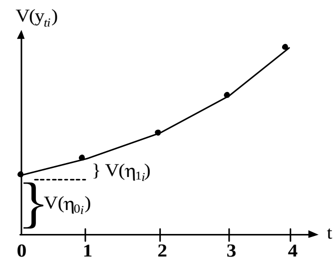
Growth Model Means And Variances

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,,$$
$$x_t = 0, 1, \ldots, T - 1.$$

Expectation (mean; E) and variance (V):

$$E\,(y_{ti}) = E\,(\eta_{0i}\,) + E\,(\eta_{1i})\, x_t\,,$$
$$V\,(y_{ti}) = V\,(\eta_{0i}\,) + V\,(\eta_{1i})\, x_t^2$$
$$+\, 2x_t\, Cov\,(\eta_{0i}\,, \eta_{1i}) + \mathrm{V}\,(\varepsilon_{ti})$$





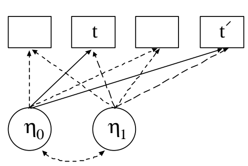
$V(\varepsilon_{ti})$ constant over t
$Cov(\eta_0\,, \eta_1) = 0$

Growth Model Covariances

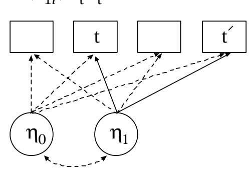
$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,,$$
$$x_t = 0, 1, \ldots, T - 1.$$

$$Cov(y_{ti}\,, y_{t'i}\,) = V(\eta_{0i}) + V(\eta_{1i})\, x_t\, x_{t'}$$
$$+\, Cov(\eta_{0i}\,, \eta_{1i})\,(x_t + x_{t'})$$
$$+\, Cov(\varepsilon_{ti}\,, \varepsilon_{t'i}\,).$$

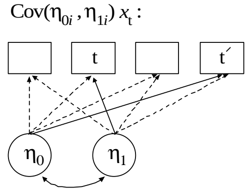
$V(\eta_{0t})$:



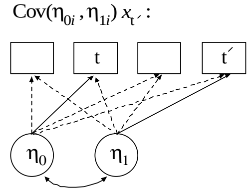
$V(\eta_{1i})\, x_t\, x_{t'}$:



$Cov(\eta_{0i}\,, \eta_{1i})\, x_t$:



$Cov(\eta_{0i}\,, \eta_{1i})\, x_{t'}$:



Growth Model Estimation, Testing, And Model Modification

- Estimation: Model parameters
  - Maximum-likelihood (ML) estimation under normality
  - ML and non-normality robust s.e.'s
  - Quasi-ML (MUML): clustered data (multilevel)
  - WLS: categorical outcomes
  - ML-EM: missing data, mixtures
- Model Testing
  - Likelihood-ratio chi-square testing; robust chi square
  - Root mean square of approximation (RMSEA): Close fit ($\leq .05$)
- Model Modification
  - Expected drop in chi-square, EPC
- Estimation: Individual growth factor values (factor scores)
  - Regression method – Bayes modal – Empirical Bayes
  - Factor determinacy

51

Alternative Growth Model Parameterizations

Parameterization 1 – for continuous outcomes

$$y_{ti} = \mathbf{0} + \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \tag{32}$$

$$\eta_{0i} = \boldsymbol{\alpha}_0 + \zeta_{0i}\,, \tag{33}$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}\,. \tag{34}$$

Parameterization 2 – for categorical outcomes and multiple indicators

$$y_{ti} = \boldsymbol{v} + \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \tag{35}$$

$$\eta_{0i} = \mathbf{0} + \zeta_{0i}\,, \tag{36}$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}\,. \tag{37}$$

52

Alternative Growth Model Parameterizations

Parameterization 1 – for continuous outcomes

- Outcome variable intercepts fixed at zero
- Growth factor means free to be estimated

```
MODEL:    i BY y1-y4@1;
          s BY y1@0 y2@1 y3@2 y4@3;
          [y1-y4@0 i s];
```

Parameterization 2 – for categorical outcomes and multiple indicators

- Outcome variable intercepts constrained to be equal
- Intercept growth factor mean fixed at zero

```
MODEL:    i BY y1-y4@1;
          s BY y1@0 y2@1 y3@2 y4@3;
          [y1-y4] (1);
          [i@0 s];
```

53

Simple Examples Of Growth Modeling

54

Steps In Growth Modeling

- Preliminary descriptive studies of the data: means, variances, correlations, univariate and bivariate distributions, outliers, etc.
- Determine the shape of the growth curve from theory and/or data
  - Individual plots
  - Mean plot
- Consider change in variance across time
- Fit model without covariates using fixed time scores
- Modify model as needed
- Add covariates

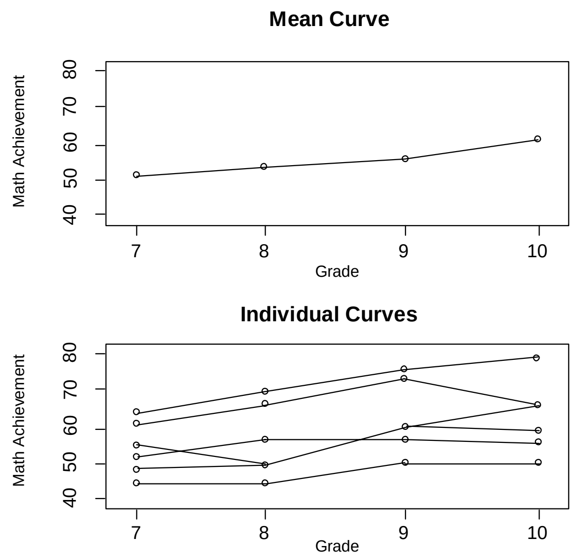
55

LSAY Data

The data come from the Longitudinal Study of American Youth (LSAY). Two cohorts were measured at four time points beginning in 1987. Cohort 1 was measured in Grades 10, 11, and 12. Cohort 2 was measured in Grades 7, 8, 9, and 10. Each cohort contains approximately 60 schools with approximately 60 students per school. The variables measured include math and science achievement items, math and science attitude measures, and background information from parents, teachers, and school principals. There are approximately 60 items per test with partial item overlap across grades – adaptive tests.

Data for the analysis include the younger females. The variables include math achievement from Grades 7, 8, 9, and 10 and the background variables of mother's education and home resources.

56



57

Input For LSAY TYPE=BASIC Analysis

```
TITLE:     LSAY For Younger Females With Listwise Deletion
           TYPE=BASIC Analysis
DATA:      FILE IS lsay.dat;
           FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10;
ANALYSIS:  TYPE = BASIC;
PLOT:      TYPE = PLOT1;
```

58

Sample Statistics For LSAY Data

n = 984

Means

	MATH7	MATH8	MATH9	MATH10
	52.750	55.411	59.128	61.796

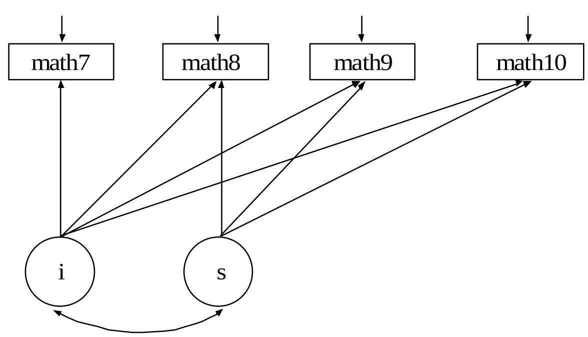
Covariances

	MATH7	MATH8	MATH9	MATH10
MATH7	81.107			
MATH8	67.663	82.829		
MATH9	73.150	76.513	100.986	
MATH10	77.952	82.668	95.158	131.326

Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	1.000			
MATH8	0.826	1.000		
MATH9	0.808	0.837	1.000	
MATH10	0.755	0.793	0.826	1.000

59



60

Input For LSAY Linear Growth Model Without Covariates

```
TITLE:     LSAY For Younger Females With Listwise Deletion
           Linear Growth Model Without Covariates
DATA:      FILE IS lsay.dat;
           FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10;
ANALYSIS:  TYPE = MEANSTRUCTURE;
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9@2 math10@3;
           [math7-math10@0];
           [i s];
OUTPUT:    SAMPSTAT STANDARDIZED MODINDICES (3.84);
```

Alternative language:

```
MODEL: i s | math7@0 math8@1 math9@2 math10@3;
```


Output Excerpts LSAY Linear Growth Model Without Covariates

Tests Of Model Fit

```
Chi-Square Test of Model Fit
          Value                        22.664
          Degrees of Freedom                5
          P-Value                      0.0004

CFI/TLI
          CFI                          0.995
          TLI                          0.994

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                     0.060
          90 Percent C.I.              0.036   0.086
          Probability RMSEA <= .05     0.223

SRMR (Standardized Root Mean Square Residual)
          Value                        0.025
```

Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Modification Indices

	M.I.	E.P.C.	Std.E.P.C.	StdYX E.P.C.
S BY MATH7	6.793	0.185	0.254	0.029
S BY MATH8	14.694	-0.169	-0.233	-0.025
S BY MATH9	9.766	0.155	0.213	0.021

63

Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
I BY					
MATH7	1.000	.000	.000	8.029	.906
MATH8	1.000	.000	.000	8.029	.861
MATH9	1.000	.000	.000	8.029	.800
MATH10	1.000	.000	.000	8.029	.708
S BY					
MATH7	.000	.000	.000	.000	.000
MATH8	1.000	.000	.000	1.377	.148
MATH9	2.000	.000	.000	2.753	.274
MATH10	3.000	.000	.000	4.130	.364

64

Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	52.623	.275	191.076	6.554	6.554
S	3.105	.075	41.210	2.255	2.255
Intercepts					
MATH7	.000	.000	.000	.000	.000
MATH8	.000	.000	.000	.000	.000
MATH9	.000	.000	.000	.000	.000
MATH10	.000	.000	.000	.000	.000

65

Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
I WITH					
S	3.491	.730	4.780	.316	.316
Residual Variances					
MATH7	14.105	1.253	11.259	14.105	.180
MATH8	13.525	.866	15.610	13.525	.156
MATH9	14.726	.989	14.897	14.726	.146
MATH10	25.989	1.870	13.898	25.989	.202
Variances					
I	64.469	3.428	18.809	1.000	1.000
S	1.895	.322	5.894	1.000	1.000

R-Square

Observed Variable	R-Square
MATH7	0.820
MATH8	0.844
MATH9	0.854
MATH10	0.798

66

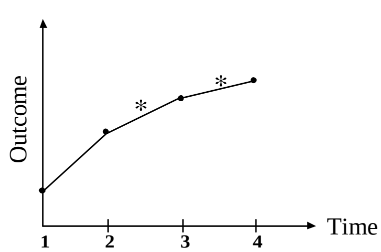
Growth Model With Free Time Scores

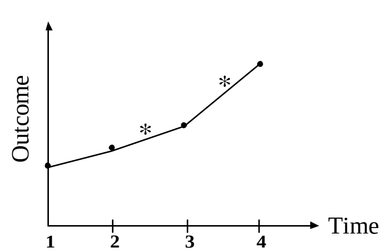
67

Specifying Time Scores For Non-Linear Growth Models With Estimated Time Scores

Non-linear growth models with estimated time scores

- Need two latent variables to describe a non-linear growth model: Intercept and slope





Time scores: 0 1 Estimated Estimated

68

Interpretation Of Slope Growth Factor Mean For Non-Linear Models

- The slope growth factor mean is the change in the outcome variable for a one unit change in the time score
- In non-linear growth models, the time scores should be chosen so that a one unit change occurs between timepoints of substantive interest.
  - An example of 4 timepoints representing grades 7, 8, 9, and 10
    - Time scores of 0 1 * * – slope factor mean refers to change between grades 7 and 8
    - Time scores of 0 * * 1 – slope factor mean refers to change between grades 7 and 10

69

Growth Model With Free Time Scores

- Identification of the model – for a model with two growth factors, at least one time score must be fixed to a non-zero value (usually one) in addition to the time score that is fixed at zero (centering point)
- Interpretation—cannot interpret the mean of the slope growth factor as a constant rate of change over all timepoints, but as the rate of change for a time score change of one.
- Approach—fix the time score following the centering point at one
- Choice of time score starting values if needed

Means	52.75	55.41	59.13	61.80
Differences	2.66	3.72		2.67
Time scores	0	1	>2	>2+1

70

Input Excerpts For LSAY Linear Growth Model With Free Time Scores Without Covariates

```
MODEL:     i s | math7@0 math8@1 math9 math10;

OUTPUT:    RESIDUAL;
```

Alternative language:

```
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9 math10;
           [math7-math10@0];
           [i s];
```

71

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates

n = 984

Tests Of Model Fit

```
Chi-Square Test of Model Fit
             Value                               4.222
             Degrees of Freedom                      3
             P-Value                            0.2373
CFI/TLI
             CFI                                 1.000
             TLI                                 0.999
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                            0.020
             90 Percent C.I.                     0.000  0.061
             Probability RMSEA <= .05            0.864
SRMR (Standardized Root Mean Square Residual)
             Value                               0.015
```

72

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I |					
MATH7	1.000	.000	.000	8.029	.903
MATH8	1.000	.000	.000	8.029	.870
MATH9	1.000	.000	.000	8.029	.797
MATH10	1.000	.000	.000	8.029	.708
S |					
MATH7	.000	.000	.000	.000	.000
MATH8	1.000	.000	.000	1.134	.123
MATH9	**2.452**	**.133**	**18.442**	**2.780**	**.276**
MATH10	**3.497**	**.199**	**17.540**	**3.966**	**.350**

73

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
S WITH					
I	3.110	.600	5.186	.342	.342
Variances					
I	64.470	3.394	18.994	1.000	1.000
S	**1.286**	**.265**	**4.853**	**1.000**	**1.000**
Means					
I	52.785	.283	186.605	6.574	6.574
S	**2.586**	**.167**	**15.486**	**2.280**	**2.280**

74

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Residuals

Model Estimated Means/Intercepts/Thresholds

MATH7	MATH8	MATH9	MATH10
52.785	55.370	59.123	61.827

Residuals for Means/Intercepts/Thresholds

MATH7	MATH8	MATH9	MATH10
-.035	.041	.004	-.031

75

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Model Estimated Covariances/Correlations/Residual Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	79.025			
MATH8	67.580	85.180		
MATH9	72.094	78.356	101.588	
MATH10	75.346	82.952	93.994	128.477

Residuals for Covariances/Correlations/Residual Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	1.999			
MATH8	.014	-2.436		
MATH9	.981	-1.921	-.705	
MATH10	2.527	-.368	1.067	2.715

76

Covariates In The Growth Model

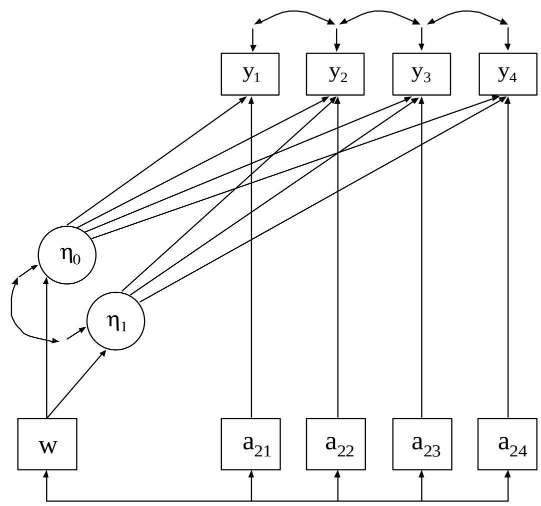
77

Covariates In The Growth Model

- Types of covariates
  - Time-invariant covariates—vary across individuals not time, explain the variation in the growth factors
  - Time-varying covariates—vary across individuals and time, explain the variation in the outcomes beyond the growth factors

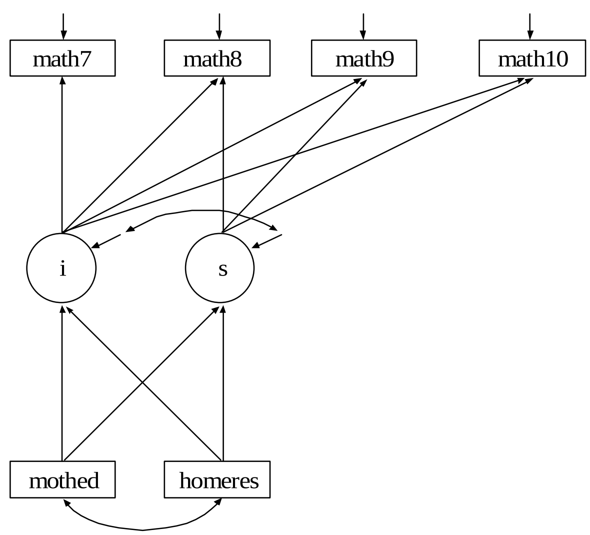
78

Time-Invariant And Time-Varying Covariates



79

LSAY Growth Model With Time-Invariant Covariates



80

Input Excerpts For LSAY Linear Growth Model With Free Time Scores And Covariates

```
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10 mothed homeres;
ANALYSIS:  !ESTIMATOR = MLM;
MODEL:     i s | math7@0 math8@1 math9 math10;
           i s ON mothed homeres;
```

Alternative language:

```
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9 math10;
           [math7-math10@0];
           [i s];
           i s ON mothed homeres;
```

81

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates

n = 935

Tests Of Model Fit for ML

```
Chi-Square Test of Model Fit
             Value                                15.845
             Degrees of Freedom                        7
             P-Value                              0.0265
CFI/TLI
             CFI                                   0.998
             TLI                                   0.995
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                              0.037
             90 Percent C.I.                       0.012  0.061
             Probability RMSEA <= .05              0.794
SRMR (Standardized Root Mean Square Residual)
             Value                                 0.015
```

82

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

Tests Of Model Fit for MLM

```
Chi-Square Test of Model Fit
             Value                               8.554*
             Degrees of Freedom                      7
             P-Value                            0.2862
             Scaling Correction Factor           1.852
                 for MLM
CFI/TLI
             CFI                                 0.999
             TLI                                 0.999
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                            0.015
SRMR (Standardized Root Mean Square Residual)
             Value                               0.015
WRMR (Weighted Root Mean Square Residual)
             Value                               0.567
```

83

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

Selected Estimates For ML

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	**2.054**	**.281**	**7.322**	**.257**	**.247**
HOMERES	**1.376**	**.182**	**7.546**	**.172**	**.255**
S ON					
MOTHED	.103	.068	1.524	.094	.090
HOMERES	**.149**	**.045**	**3.334**	**.136**	**.201**
I WITH					
S	2.604	.559	4.658	.297	.297
Residual Variances					
I	53.931	2.995	18.008	.842	.842
S	1.134	.253	4.488	.942	.942
Intercepts					
I	43.877	.790	55.531	5.484	5.484
S	1.859	.221	8.398	1.695	1.695

84

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

R-Square

Observed Variable	R-Square
MATH7	0.813
MATH8	0.849
MATH9	0.861
MATH10	0.796

Latent Variable	R-Square
I	.158
S	.058

85

Model Estimated Average And Individual Growth Curves With Covariates

Model:

$$y_{ti} = \eta_{0i} + \eta_{1i} \, x_t + \varepsilon_{ti} \, , \qquad (23)$$

$$\eta_{0i} = \alpha_0 + \gamma_0 \, w_i + \zeta_{0i} \, , \qquad (24)$$

$$\eta_{1i} = \alpha_1 + \gamma_1 \, w_i + \zeta_{1i} \, , \qquad (25)$$

Estimated growth factor means:

$$\hat{E}(\eta_{0i}) = \hat{\alpha}_0 + \hat{\gamma}_0 \overline{w} \, , \qquad (26)$$

$$\hat{E}(\eta_{1i}) = \hat{\alpha}_1 + \hat{\gamma}_1 \overline{w} \; . \qquad (27)$$

Estimated outcome means:

$$\hat{E}(y_{ti}) = \hat{E}(\eta_{0i}) + \hat{E}(\eta_{1i}) \, x_t \, . \qquad (28)$$

Estimated outcomes for individual *i* :

$$\hat{y}_{ti} = \hat{\eta}_{0i} + \hat{\eta}_{1i} \; x_t \qquad (29)$$

where $\hat{\eta}_{0i}$ and $\hat{\eta}_{1i}$ are estimated factor scores. $\hat{y}_{ti}$ can be used for prediction purposes.

86

Model Estimated Means With Covariates

Model estimated means are available using the TECH4 and RESIDUAL options of the OUTPUT command.

Estimated Intercept Mean = Estimated Intercept +
Estimated Slope (Mothed)*
Sample Mean (Mothed) +
Estimated Slope (Homeres)*
Sample Mean (Homeres)

43.88 + 2.05*2.31 + 1.38*3.11 = 52.9

Estimated Slope Mean = Estimated Intercept +
Estimated Slope (Mothed)*
Sample Mean (Mothed) +
Estimated Slope (Homeres)*
Sample Mean (Homeres)

1.86 + .10*2.31 + .15*3.11 = 2.56

87

Model Estimated Means With Covariates (Continued)

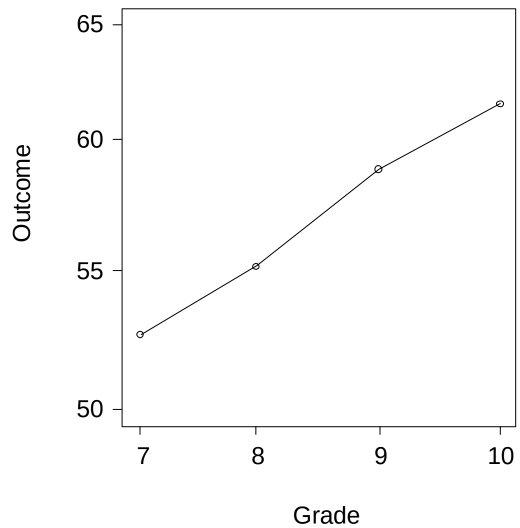
Estimated Outcome Mean at Timepoint t =

Estimated Intercept Mean +
Estimated Slope Mean * (Time Score at Timepoint t)

Estimated Outcome Mean at Timepoint 1 =
52.9 + 2.56 * (0) = **52.9**
Estimated Outcome Mean at Timepoint 2 =
52.9 + 2.56 * (1.00) = **55.46**
Estimated Outcome Mean at Timepoint 3 =
52.9 + 2.56 * (2.45) = **59.17**
Estimated Outcome Mean at Timepoint 4 =
52.9 + 2.56 * (3.50) = **61.86**

88

Estimated LSAY Curve



89

Centering

90

Centering

- Centering determines the interpretation of the intercept growth factor
- The centering point is the timepoint at which the time score is zero
- A model can be estimated for different centering points depending on which interpretation is of interest
- Models with different centering points give the same model fit because they are reparameterizations of the model
- Changing the centering point in a linear growth model with four timepoints

Timepoints	1	2	3	4	
					Centering at
Time scores	0	1	2	3	Timepoint 1
	-1	0	1	2	Timepoint 2
	-2	-1	0	1	Timepoint 3
	-3	-2	-1	0	Timepoint 4

Input Excerpts For LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10

```
MODEL:     i s | math7*-3 math8*-2 math9@-1 math10@0;
           i s ON mothed homeres;
```

Alternative language:

```
MODEL:     i BY math7-math10@1;
           s BY math7*-3 math8*-2 math9@-1 math10@0;
           [math7-math10@0];
           [i s];
           i s ON mothed homeres;
```

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10

n = 935

Tests of Model Fit

```
CHI-SQUARE TEST OF MODEL FIT

          Value                         15.845
          Degrees of Freedom                 7
          P-Value                        .0265

RMSEA (ROOT MEAN SQUARE ERROR OF APPROXIMATION)

          Estimate                       .037
          90 Percent C.I.                .012    .061
          Probability RMSEA <= .05       .794
```

93

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10 (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	2.418	0.353	6.851	0.238	0.229
HOMERES	1.903	0.229	8.294	0.187	0.277
S ON					
MOTHED	0.111	0.073	1.521	0.094	0.090
HOMERES	0.161	0.049	3.311	0.136	0.201

94

Further Readings On Introductory Growth Modeling

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Further Practical Issues