Further Readings On Factor Analysis Of Categorical Outcomes

Bock, R.D., Gibbons, R., & Muraki, E.J. (1998). Full information item factor analysis. Applied Psychological Measurement, 12, 261-280.

Flora, D.B. & Curran, P.J., (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. Psychological Methods, 9, 466-491.

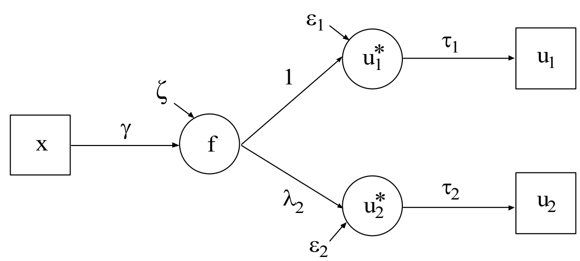
Muthén, B. (1989). Dichotomous factor analysis of symptom data. In Eaton & Bohrnstedt (Eds.), Latent variable models for dichotomous outcomes: Analysis of data from the epidemiological Catchment Area program (pp.19-65), a special issue of Sociological Methods & Research, 18, 19-65.

Muthen, B. & Kaplan, D. (1985). A comparison of some methodologies for the factor analysis of non-normal Likert variables. British Journal of Mathematical and Statistical Psychology, 38, 171-189.

Muthen, B. & Kaplan, D. (1992). A comparison of some methodologies for the factor analysis of non-normal Likert variables: A note on the size of the model. British Journal of Mathematical and Statistical Psychology, 45, 19-30.

CFA With Covariates (MIMIC)

CFA With Covariates Using WLS



$u^*_{ij} = \lambda_j f_i + \varepsilon_{ij}$, $(j = 1, 2)$

$f_i = \gamma\, x_i + \zeta_i$

Estimate CFA model by fitting to probit / logit regression estimates

143

CFA With Covariates (MIMIC)

Used to study the effects of covariates or background variables on the factors and outcome variables to understand measurement invariance and heterogeneity

- Measurement non-invariance – direct relationships between the covariates and outcome variables that are not mediated by the factors – if they are significant, this indicates measurement non-invariance due to differential item functioning (DIF)
- Population heterogeneity – relationships between the covariates and the factors – if they are significant, this indicates that the factor means are different for different levels of the covariates.

Model Assumptions

- Same factor loadings and observed residual variances / covariances for all levels of the covariates
- Same factor variances and covariances for all levels of the covariates

144

Steps In CFA With Covariates

- Establish a CFA or EFA/CFA model
- Add covariates – check that factor structure does not change and study modification indices for possible direct effects
- Add direct effects suggested by modification indices – check that factor structure does not change
- Interpret the model
  - Factors
  - Effects of covariates on factors
  - Effects of covariates on factor indicators

Antisocial Behavior (ASB) Data

The Antisocial Behavior (ASB) data were taken from the National Longitudinal Survey of Youth (NLSY) that is sponsored by the Bureau of Labor Statistics. These data are made available to the public by Ohio State University. The data were obtained as a multistage probability sample with oversampling of blacks, Hispanics, and economically disadvantaged non-blacks and non-Hispanics.

Data for the analysis include 15 of the 17 antisocial behavior items that were collected in 1980 when respondents were between the ages of 16 and 23 and the background variables of age, gender and ethnicity. The ASB items assessed the frequency of various behaviors during the past year. A sample of 7,326 respondents has complete data on the antisocial behavior items and the background variables of age, gender, and ethnicity. Following is a list of the 15 items:

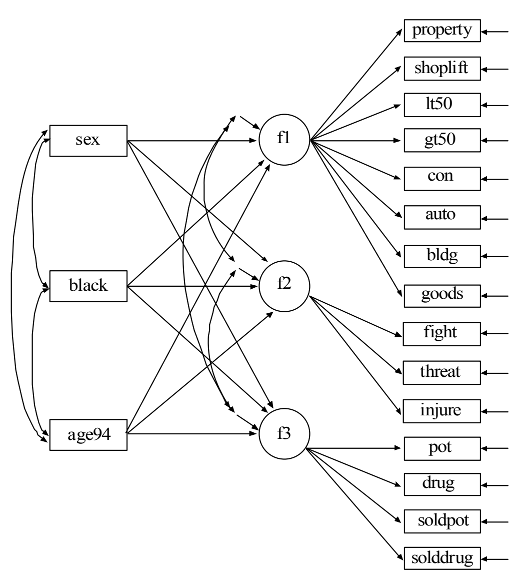
Antisocial Behavior (ASB) Data (Continued)

Damaged property
Fighting
Shoplifting
Stole < $50
Stole > $50
Seriously threaten
Intent to injure
Use marijuana

Use other drugs
Sold marijuana
Sold hard drugs
“Con” someone
Take auto
Broken into building
Held stolen goods

These items were dichotomized 0/1 with 0 representing never in the last year. An EFA suggested three factors: property offense, person offense, and drug offense.

147



148

Input For CFA With Covariates With Categorical Outcomes For 15 ASB Items

```
TITLE:      CFA with covariates with categorical outcomes using
            15 antisocial behavior items and 3 covariates

DATA:       FILE IS asb.dat;
            FORMAT IS 34X 54F2.0;

VARIABLE:   NAMES ARE property fight shoplift lt50 gt50 force
            threat injure pot drug soldpot solddrug con auto bldg
            goods gambling dsm1-dsm22 sex black hisp single
            divorce dropout college onset fhist1 fhist2 fhist3
            age94 cohort dep abuse;

            USEV ARE property-gt50 threat-goods sex black age94;

            CATEGORICAL ARE property-goods;
```

149

Input For CFA With Covariates With Categorical Outcomes For 15 ASB Items (Continued)

```
MODEL:      f1 BY property shoplift-gt50 con-goods;

            f2 BY fight threat injure;

            f3 BY pot-solddrug;

            f1-f3 ON sex black age94;

            property-goods ON sex-age94@0;

OUTPUT:     STANDARDIZED MODINDICES;
```

150

Output Excerpts CFA With Covariates With Categorical Outcomes For 15 ASB Items

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
F1 BY					
PROPERTY	1.000	.000	.000	.791	.760
SHOPLIFT	.974	.023	42.738	.771	.742
LT50	.915	.023	39.143	.724	.700
GT50	1.055	.031	33.658	.835	.799
CON	.752	.024	31.637	.595	.581
AUTO	.796	.030	26.462	.629	.613
BLDG	1.084	.030	35.991	.858	.818
GOODS	1.071	.025	42.697	.847	.809

151

Output Excerpts CFA With Covariates With Categorical Outcomes For 15 ASB Items (Continued)

F2 BY					
FIGHT	1.000	.000	.000	.773	.734
THREAT	1.096	.035	31.382	.847	.797
INJURE	1.082	.037	28.888	.836	.787
F3 BY					
POT	1.000	.000	.000	.866	.851
DRUG	1.031	.023	45.818	.893	.876
SOLDPOT	1.046	.023	45.844	.905	.888
SOLDDRUG	.923	.036	25.684	.799	.787

152

Output Excerpts CFA With Covariates With Categorical Outcomes For 15 ASB Items (Continued)

```
F1          ON
   SEX                 .516       .024    21.206       .653       .326
   BLACK              -.080       .025    -3.168      -.102      -.047
   AGE94              -.054       .006    -9.856      -.069      -.150

F2          ON
   SEX                 .561       .026    21.715       .726       .363
   BLACK               .174       .025     7.087       .225       .103
   AGE94              -.068       .006   -12.286      -.087      -.191

F3          ON
   SEX                 .229       .026     8.760       .265       .132
   BLACK              -.272       .029    -9.384      -.315      -.144
   AGE94               .039       .006     6.481       .045       .099
```

153

Output Excerpts CFA With Covariates With Categorical Outcomes For 15 ASB Items (Continued)

Tests Of Model Fit

```
Chi-Square Test of Model Fit
            Value                      1225.266*
            Degrees of Freedom              105**
            P-Value                       0.0000
CFI / TLI
            CFI                            0.945
            TLI                            0.964
RMSEA (Root Mean Square Error Of Approximation)
            Estimate                       0.038
WRMR (Weighted Root Mean Square Residual)
            Value                          2.498
```

154

Output Excerpts CFA With Covariates With Categorical Outcomes For 15 ASB Items (Continued)

Modification Indices

PROPERTY ON BLACK	4.479	GT50 ON SEX	12.100
PROPERTY ON AGE94	28.229	GT50 ON BLACK	12.879
FIGHT ON SEX	60.599	GT50 ON AGE94	7.413
FIGHT ON BLACK	26.695	THREAT ON SEX	10.221
FIGHT ON AGE94	**64.815**	THREAT ON BLACK	26.665
SHOPLIFT ON SEX	**131.792**	THREAT ON AGE94	3.892
SHOPLIFT ON BLACK	0.039	INJURE ON SEX	22.803
SHOPLIFT ON AGE94	0.038	INJURE ON BLACK	0.089
LT50 ON SEX	0.040	INJURE ON AGE94	42.549
LT50 ON BLACK	22.530	POT ON SEX	10.727
LT50 ON AGE94	24.750	POT ON BLACK	12.177
		POT ON AGE94	17.432

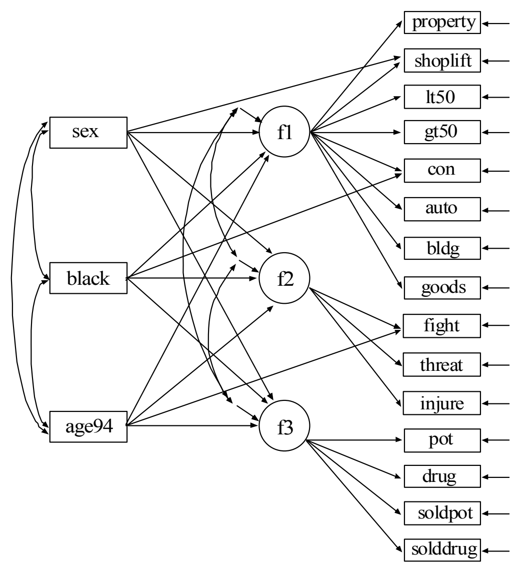
155

Output Excerpts CFA With Covariates With Categorical Outcomes For 15 ASB Items (Continued)

Modification Indices

DRUG ON SEX	15.637	AUTO ON SEX	0.735
DRUG ON BLACK	41.202	AUTO ON BLACK	1.414
DRUG ON AGE94	1.583	AUTO ON AGE94	2.936
SOLDPOT ON SEX	51.496	BLDG ON SEX	37.797
SOLDPOT ON BLACK	1.242	BLDG ON BLACK	7.053
SOLDPOT ON AGE94	29.267	BLDG IB AGE94	0.114
SOLDDRUG ON SEX	3.920	GOODS ON SEX	24.664
SOLDDRUG ON BLACK	7.187	GOODS ON BLACK	0.982
SOLDDRUG ON AGE94	2.956	GOODS ON AGE94	6.061
CON ON SEX	31.521		
CON ON BLACK	**80.515**		
CON ON AGE94	11.259		

156



157

Input Excerpts For ASB CFA With Covariates And Direct Effects

```
MODEL:      f1 BY property shoplift-gt50 con-goods;
            f2 BY fight threat injure;
            f3 BY pot-solddrug;

            f1-f3 ON sex black age94;

            shoplift ON sex;
            con ON black;
            fight ON age94;
```

158

Input Excerpts For ASB CFA With Covariates And Direct Effects (Continued)

Tests Of Model Fit

```
Chi-Square Test of Model Fit
              Value                                946.256 *
              Degrees of Freedom                       102 **
              P-Value                               0.0000
CFI/TLI
              CFI                                    0.959
              TLI                                    0.972
RMSEA (Root Mean Square Error Of Approximation)
              Estimate                               0.034
WRMR (Weighted Root Mean Square Residual)
              Value                                  2.198
```


Output Excerpts For ASB CFA With Covariates And Direct Effects (Continued)

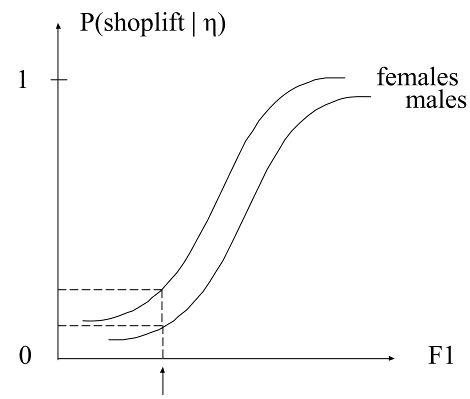
```
                    Estimates     S.E.     Est./S.E.     Std       StdYX

F1          BY
   SHOPLIFT         1.002         .024      42.183       .805      .793

F1          ON
   SEX               .596         .026      22.958       .742      .371

SHOPLIFT    ON
   SEX              -.385         .033     -11.594      -.385     -.190
CON         ON
   BLACK             .305         .034       8.929       .305      .136
FIGHT       ON
   AGE94            -.068         .008      -8.467      -.068    - .138

Thresholds

   SHOPLIFT$1         .558        .033      17.015       .558       .558

R-SQUARE

    Observed    Residual
    Variable    Variance   R-Square

    SHOPLIFT       .461       .552
```

Interpretation Of Direct Effects

Shoplift On Gender

- Indirect effect of gender on shoplift
  - F1 has a positive relationship with gender – males have a higher mean than females on the f1 factor
  - Shoplift has a positive loading on the f1 factor
  - Conclusion: males are expected to have a higher probability of shoplifting
- Effect of gender on shoplift
  - Direct effect is negative – for a given factor value, males have a lower probability of shoplifting than females
  - Conclusion – shoplift is not invariant

Calculating Item Probabilities



Graph can be done in Mplus using the PLOT command and the option "Item characteristic curves".

Calculating Item Probabilities (Continued)

The model with a direct effect from x to item u_j,

$$u^*_{ij} = \lambda_j \eta_i + \kappa_j x_i + \varepsilon_{ij}, \qquad (45)$$

gives the conditional probability of a $u = 1$ response given the factor η_i and the covariate x_i

$$P(u_{ij} = 1 \mid \eta_i, x_i) = 1 - F[(\tau_j - \lambda_j \eta_i - \kappa_j x_i)\, \theta_{jj}^{-1/2}], \qquad (46)$$

$$= F[(-\tau_j + \lambda_j \eta_i + \kappa_j x_i)\, \theta_{jj}^{-1/2}], \qquad (47)$$

where F is the normal distribution function and θ is the residual variance.

For example, for the item shoplift, $\tau_j = 0.558$, $\kappa_j = -0.385$, $\theta_{jj} = 0.461$. At $\eta = 0$, the probability is 0.21 for females ($x = 0$) and 0.08 for males ($x = 1$).

Calculating Item Probabilities (Continued)

Consider

$$P(u_{ij} = 1 \mid \eta_{ij}, x_i) = 1 - F[(\tau_j - \lambda_j \eta_i - \kappa_j x_i)\, \theta_{jj}^{-1/2}], \qquad (47)$$

using $\tau_j = 0.558$, $\kappa_j = -0.385$, $\theta_{jj} = 0.461$, and $\eta = 0$.

Here, $\theta_{jj}^{-1/2} = \frac{1}{\sqrt{\theta_{jj}}} = \frac{1}{\sqrt{0.461}} = 1.473$.

For females ($x = 0$):

1. $(\tau_j - \lambda_j \eta_i - \kappa_j x_i) = 0.558 - 1.002 \times 0 - (-0.385) \times 0 = 0.558$.
2. $(\tau_j - \lambda_j \eta_i - \kappa_j x_i)\, \theta_{jj}^{-1/2} = 0.558 \times 1.473 = 0.822$.
3. $F[0.822] = 0.794$ using a z table
4. $1 - 0.794 = 0.206$.

For males ($x = 1$):

1. $(\tau_j - \lambda_j \eta_i - \kappa_j x_i) = 0.558 - 1.002 \times 0 - (-0.385) \times 1 = 0.943$.
2. $(\tau_j - \lambda_j \eta_i - \kappa_j x_i)\, \theta_{jj}^{-1/2} = 0.943 \times 1.473 = 1.389$.
3. $F[1.389] = 0.918$ using a z table.
4. $1 - 0.918 = 0.082$.

Further Readings On Factor Analysis And MIMIC Analysis With Categorical Outcomes

Gallo, J.J., Anthony, J. & Muthen, B. (1994). Age differences in the symptoms of depression: a latent trait analysis. Journals of Gerontology: Psychological Sciences, 49, 251-264. (#52)

Mislevy, R. (1986). Recent developments in the factor analysis of categorical variables. Journal of Educational Statistics, 11, 3-31.

Muthén, B. (1978). Contributions to factor analysis of dichotomous variables. Psychometrika, 43, 551-560. (#3)

Muthén, B. (1989). Dichotomous factor analysis of symptom data. In Eaton & Bohrnstedt (Eds.), Latent variable models for dichotomous outcomes: Analysis of data from the Epidemiological Catchment Area Program (pp. 19-65), a special issue of Sociological Methods & Research, 18, 19-65. (#21)

Further Readings On Factor Analysis And MIMIC Analysis With Categorical Outcomes (Continued)

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Muthén, B., Tam, T., Muthén, L., Stolzenberg, R. M., & Hollis, M. (1993). Latent variable modeling in the LISCOMP framework: Measurement of attitudes toward career choice. In D. Krebs, & P. Schmidt (Eds.), New directions in attitude measurement, Festschrift for Karl Schuessler (pp. 277-290). Berlin: Walter de Gruyter. (#46)

Multiple Group Analysis With Categorical Outcomes

167

Steps In Multiple Group Analysis

- Fit the model separately in each group
- Fit the model in all groups allowing all parameters to be free except factor means which are fixed to zero in all groups and scale factors which are fixed to one in all groups
- Fit the model in all groups holding factor loadings and thresholds equal across groups with factor means fixed to zero in the first group and free in the other groups and scale factors fixed to one in the first group and free in the other groups
- Add covariates
- Modify the model

168

Inputs For Multiple Group Analysis Of 15 ASB Items

Measurement Non-Invariance

```
MODEL:       f1 BY property shoplift-gt50 con-goods;
             f2 BY fight threat injure;
             f3 BY pot-solddrug;
             [f1-f3@0];
             {property-goods@1};
MODEL male:
             f1 BY shoplift-gt50 con-goods;
             f2 BY threat injure;
             f3 BY drug-solddrug;
             [property$1-goods$1];
```

169

Inputs For Multiple Group Analysis Of 15 ASB Items (Continued)

Measurement Invariance

```
MODEL:       f1 BY property shoplift-gt50 con-goods;
             f2 BY fight threat injure;
             f3 BY pot-solddrug;
```

Partial Measurement Invariance

```
MODEL:       f1 BY property shoplift-gt50 con-goods;
             f2 BY fight* threat@1 injure;
             f3 BY pot-solddrug;
MODEL male:
             f1 BY con lt50;
             f2 BY fight;
             f3 BY soldpot pot solddrug;
             [con$1 lt50$1 fight$1 soldpot$1 pot$1 solddrug$1];
             {con@1 lt50@1 fight@1 soldpot@1 pot@1 solddrug@1};
```

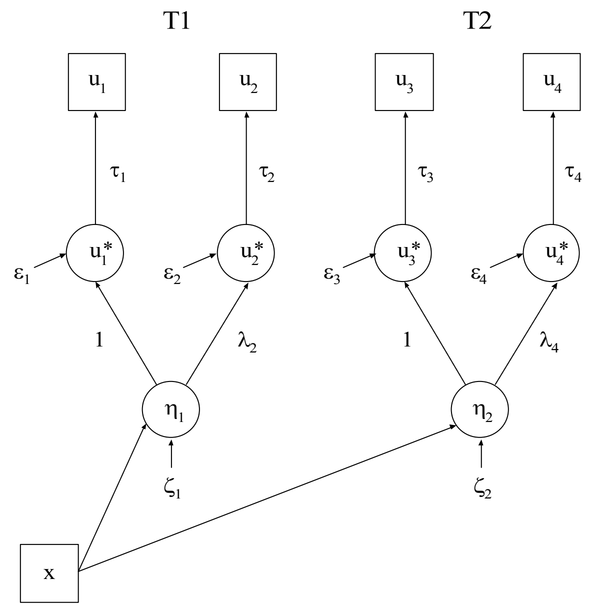
170

Further Readings On Multiple-Group Analysis Of Categorical Outcomes

Muthén, B. & Asparouhov, T. (2002). Latent variable analysis with categorical outcomes: Multiple-group and growth modeling in Mplus. Mplus Web Note #4 (www.statmodel.com).

Muthén, B., & Christoffersson, A. (1981). Simultaneous factor analysis of dichotomous variables in several groups. Psychometrika, 46, 407-419. (#6)

Technical Issues For Weighted-Least Squares Estimation



173

Latent Response Variable Modeling

- The analysis considers means (thresholds) and correlations because variances do not contribute further information
  - $E(u) = \pi$, $V(u) = \pi\,(1 - \pi)$
- For each u (see figure)
  - Normality of u^* given x (probit)
  - Residual variance fixed at 1 implies $V(\varepsilon)$ not free,

$$V(u^* \mid x) = \lambda^2\, V(\zeta) + V(\varepsilon) = 1, \qquad (8)$$

$$i.e.\ \ V(\varepsilon) = 1 - \lambda^2\, V(\zeta) \qquad (9)$$

- For pairs of u's
  - Multivariate normal u^*'s given x
  - Because residual variances are one, u^* residual correlations are considered, not covariances
  - Normality of u^*'s given x is less strong than normal u^* and normal x, assumed for polychoric and polyserial correlations

174

Scale Factors With Measurement Invariance

Problem: Correlations should not be used when comparing relationships for variables with different variances.

Solution: Add scale factors δ to the model, $\delta = 1/\sqrt{V(u^* \mid x)}$.

Example (see figure): Aim is to test measurement invariance, e.g. $\tau_2 = \tau_4 = \tau$, $\lambda_2 = \lambda_4 = \lambda$.

$$V(u_2^* \mid x) = \lambda^2\, V(\zeta_1) + V(\varepsilon_2), \quad (40)$$

$$V(u_4^* \mid x) = \lambda^2\, V(\zeta_2) + V(\varepsilon_4), \quad (41)$$

showing that $V(u^* \mid x)$ varies across the two variables if either $V(\zeta)$ or $V(\varepsilon)$ varies, even though λ is invariant.

Fixing both $V(u_2^* \mid x)$ and $V(u_4^* \mid x)$ to 1 is therefore wrong under measurement invariance. Instead, use

$$\delta_2 = 1, \quad (42)$$

$$\delta_4 \text{ free.} \quad (43)$$

By letting δ_4 be free, the model allows $V(u_4^* \mid x) \neq V(u_2^* \mid x)$, while still modeling the u_2^*, u_4^* correlation

$$Cov(u_2^*, u_4^* \mid x)\, \delta_2\, \delta_4 . \quad (44)$$

175

Estimation With Categorical Outcomes

Full information maximum-likelihood estimation is heavy for general models.

Limited-information weighted least squares:

Fitting function:

$WLS = 1/2\ (\boldsymbol{s} - \boldsymbol{\sigma})' \boldsymbol{W}^{-1} (\boldsymbol{s} - \boldsymbol{\sigma})$

Sample statistics:

- $\boldsymbol{s}_1$: probit thresholds
- $\boldsymbol{s}_2$: probit regression slopes ($q > 0$)
- $\boldsymbol{s}_3$: probit residual correlations
- $\boldsymbol{s}' = (\boldsymbol{s}'_1, \boldsymbol{s}'_2, \boldsymbol{s}'_3)$

Weight matrix:

- Full $\boldsymbol{W}$ (GLS/WLS: $\boldsymbol{W}$ = asympt $V(\boldsymbol{s})$)
- Diagonal $\boldsymbol{W}$ (WLSM, WLSMV)

Robust standard errors and chi-square in line with Satorra

176

Further Readings On Technical Aspects Of Weighted Least Squares With Categorical Outcomes

Muthén, B. (1984). A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators. Psychometrika, 49, 115-132. (#11)

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Muthén, B. & Satorra, A. (1995). Technical aspects of Muthén's LISCOMP approach to estimation of latent variable relations with a comprehensive measurement model. Psychometrika, 60, 489-503.

Muthén, B. du Toit, S.H.C. & Spisic, D. (1997). Robust inference using weighted least squares and quadratic estimating equations in latent variable modeling with categorical and continuous outcomes. Accepted for publication in Psychometrika. (#75)

177

Levels Of Engagement

- Mplus support for licensed Mplus users
- Mplus Discussion for brief Mplus analysis questions of general interest
- Statistical consulting not available through Mplus
- Research interaction on topics of common interest
- SEMNET

178

References

Analysis With Categorical Outcomes

General

Agresti, A. (2002). Categorical data analysis. Second edition. New York: John Wiley & Sons.

Agresti, A. (1996). An introduction to categorical data analysis. New York: Wiley.

Hosmer, D.W. & Lemeshow, S. (2000). Applied logistic regression. Second edition. New York: John Wiley & Sons.

McKelvey, R.D. & Zavoina, W. (1975). A statistical model for the analysis of ordinal level dependent variables. Journal of Mathematical Sociology, 4, 103-120.

Censored and Poisson Regression

Hilbe, J. M. (2007). Negative binomial regression. Cambridge, UK: Cambridge University Press.

Lambert, D. (1992). Zero-inflated Poisson regression, with an application to defects in manufacturing. Technometrics, 34, 1-13.

References (Continued)

Long, S. (1997). Regression models for categorical and limited dependent variables. Thousand Oaks: Sage.

Maddala, G.S. (1983). Limited-dependent and qualitative variables in econometrics. Cambridge: Cambridge University Press.

Tobin, J (1958). Estimation of relationships for limited dependent variables. Econometrica, 26, 24-36.

IRT

Baker, F.B. & Kim, S.H. (2004). Item response theory. Parameter estimation techniques. Second edition. New York: Marcel Dekker.

Bock, R.D. (1997). A brief history of item response theory. Educational Measurement: Issues and Practice, 16, 21-33.

du Toit, M. (2003). IRT from SSI. Lincolnwood, IL: Scientific Software International, Inc. (BILOG, MULTILOG, PARSCALE, TESTFACT)

Embretson, S. E., & Reise, S. P. (2000). Item response theory for psychologists. Mahwah, NJ: Erlbaum.

Hambleton, R.K. & Swaminathan, H. (1985). Item response theory. Boston: Kluwer-Nijhoff.

MacIntosh, R. & Hashim, S. (2003). Variance estimation for converting MIMIC model parameters to IRT parameters in DIF analysis. Applied Psychological Measurement, 27, 372-379.

References (Continued)

Muthén, B. (1985). A method for studying the homogeneity of test items with respect to other relevant variables. Journal of Educational Statistics, 10, 121-132. (#13)

Muthén, B. (1988). Some uses of structural equation modeling in validity studies: Extending IRT to external variables. In H. Wainer & H. Braun (Eds.), Test Validity (pp. 213-238). Hillsdale, NJ: Erlbaum Associates. (#18)

Muthén, B. (1989). Using item-specific instructional information in achievement modeling. Psychometrika, 54, 385-396. (#30)

Muthén, B. (1994). Instructionally sensitive psychometrics: Applications to the Second International Mathematics Study. In I. Westbury, C. Ethington, L. Sosniak & D. Baker (Eds.), In search of more effective mathematics education: Examining data from the IEA second international mathematics study (pp. 293-324). Norwood, NJ: Ablex. (#54)

Muthén, B. & Asparouhov, T. (2002). Latent variable analysis with categorical outcomes: Multiple-group and growth modeling in Mplus. Mplus Web Note #4 (www.statmodel.com).

Muthén, B., Kao, Chih-Fen & Burstein, L. (1991). Instructional sensitivity in mathematics achievement test items: Applications of a new IRT-based detection technique. Journal of Educational Measurement, 28, 1-22. (#35)

References (Continued)

Muthén, B. & Lehman, J. (1985). Multiple-group IRT modeling: Applications to item bias analysis. Journal of Educational Statistics, 10, 133-142. (#15)

Takane, Y. & DeLeeuw, J. (1987). On the relationship between item response theory and factor analysis of discretized variables. Psychometrika, 52, 393-408.

Factor Analysis

Bartholomew, D.J. (1987). Latent variable models and factor analysis. New York: Oxford University Press.

Bock, R.D., Gibbons, R., & Muraki, E.J. (1988). Full information item factor analysis. Applied Psychological Measurement, 12, 261-280.

Blafield, E. (1980). Clustering of observations from finite mixtures with structural information. Unpublished doctoral dissertation, Jyvaskyla studies in computer science, economics, and statistics, Jyvaskyla, Finland.

Flora, D.B. & Curran P.J. (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. Psychological Methods, 9, 466-491.

Lord, F.M. & Novick, M.R. (1968). Statistical theories of mental test scores. Reading, Mass.: Addison-Wesley Publishing Co.

References (Continued)

Millsap, R.E. & Yun-Tien, J. (2004). Assessing factorial invariance in ordered-categorical measures. Multivariate Behavioral Research, 39, 479-515.

Mislevy, R. (1986). Recent developments in the factor analysis of categorical variables. Journal of Educational Statistics, 11, 3-31.

Muthén, B. (1978). Contributions to factor analysis of dichotomous variables. Psychometrika, 43, 551-560. (#3)

Muthén, B. (1989). Dichotomous factor analysis of symptom data. In Eaton & Bohrnstedt (Eds.), Latent variable models for dichotomous outcomes: Analysis of data from the Epidemiological Catchment Area program (pp. 19-65), a special issue of Sociological Methods & Research, 18, 19-65. (#21)

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Muthén, B. (1996). Psychometric evaluation of diagnostic criteria: Application to a two-dimensional model of alcohol abuse and dependence. Drug and Alcohol Dependence, 41, 101-112. (#66)

Muthén, B. & Asparouhov, T. (2002). Latent variable analysis with categorical outcomes: Multiple-group and growth modeling in Mplus. Mplus Web Note #4 (www.statmodel.com).

References (Continued)

Muthén, B. & Christoffersson, A. (1981). Simultaneous factor analysis of dichotomous variables in several groups. Psychometrika, 46, 407-419. (#6)

Muthén, B. & Kaplan, D. (1985). A comparison of some methodologies for the factor analysis of non-normal Likert variables. British Journal of Mathematical and Statistical Psychology, 38, 171-189.

Muthén, B. & Kaplan, D. (1992). A comparison of some methodologies for the factor analysis of non-normal Likert variables: A note on the size of the model. British Journal of Mathematical and Statistical Psychology, 45, 19-30.

Muthén, B. & Satorra, A. (1995). Technical aspects of Muthén's LISCOMP approach to estimation of latent variable relations with a comprehensive measurement model. Psychometrika, 60, 489-503.

Takane, Y. & DeLeeuw, J. (1987). On the relationship between item response theory and factor analysis of discretized variables. Psychometrika, 52, 393-408.

Yung, Y.F. (1997). Finite mixtures in confirmatory factor-analysis models. Psychometrika, 62, 297-330.

References (Continued)

MIMIC

Gallo, J.J., Anthony, J. & Muthén, B. (1994). Age differences in the symptoms of depression: a latent trait analysis. Journals of Gerontology: Psychological Sciences, 49, 251-264. (#52)

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Muthén, B., Tam, T., Muthén, L., Stolzenberg, R.M. & Hollis, M. (1993). Latent variable modeling in the LISCOMP framework: Measurement of attitudes toward career choice. In D. Krebs & P. Schmidt (Eds.), New directions in attitude measurement, Festschrift for Karl Schuessler (pp. 277-290). Berlin: Walter de Gruyter. (#46)

SEM

Browne, M.W. & Arminger, G. (1995). Specification and estimation of mean- and covariance-structure models. In G. Arminger, C.C. Clogg & M.E. Sobel (Eds.), Handbook of statistical modeling for the social and behavioral sciences (pp. 311-359). New York: Plenum Press.

References (Continued)

MacKinnon, D.P., Lockwood, C.M., Brown, C.H., Wang, W., & Hoffman, J.M. (2007). The intermediate endpoint effect in logistic and probit regression. Clinical Trials, 4, 499-513.

Muthén, B. (1979). A structural probit model with latent variables. Journal of the American Statistical Association, 74, 807-811. (#4)

Muthén, B. (1983). Latent variable structural equation modeling with categorical data. Journal of Econometrics, 22, 48-65. (#9)

Muthén, B. (1984). A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators. Psychometrika, 49, 115-132. (#11)

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24) Muthén, B. (1993). Goodness of fit with categorical and other non-normal variables. In K.A. Bollen, & J.S. Long (Eds.), Testing structural equation models (pp. 205-243). Newbury Park, CA: Sage. (#45).

Muthén, B. & Speckart, G. (1983). Categorizing skewed, limited dependent variables: Using multivariate probit regression to evaluate the California Civil Addict Program. Evaluation Review, 7, 257-269. (#3)

Muthén, B. du Toit, S.H.C. & Spisic, D. (1997). Robust inference using weighted least squares and quadratic estimating equations in latent variable modeling with categorical and continuous outcomes. Accepted for publication in Psychometrika. (#75)

References (Continued)

Prescott, C.A. (2004). Using the Mplus computer program to estimate models for continuous and categorical data from twins. Behavior Genetics, 34, 17-40.

Xie, Y. (1989). Structural equation models for ordinal variables. Sociological Methods & Research, 17, 325-352.

Yu, C.-Y. & Muthén, B. (2002). Evaluation of model fit indices for latent variable models with categorical and continuous outcomes. Technical report.