Principal Components (PCA) & Exploratory Factor Analysis (EFA) with SPSS

IDRE Statistical Consulting

https://stats.idre.ucla.edu/spss/seminars/efa-spss/

Outline

- Introduction
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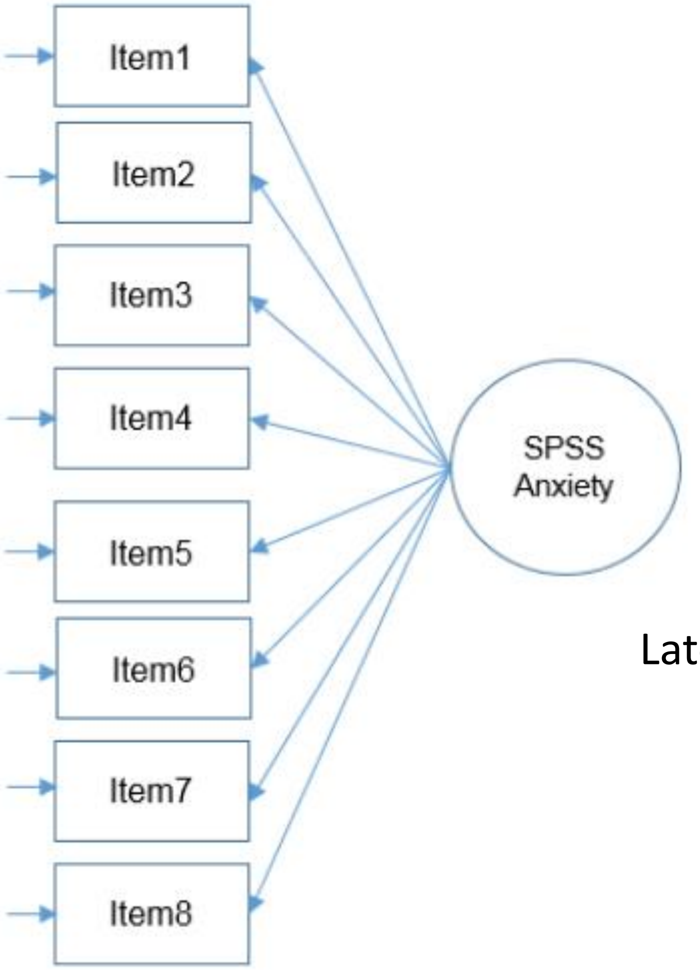
  - Pearson correlation
  - Partitioning the variance in factor analysis
- Extracting factors
  - Principal components analysis
    - Running a PCA with 8 components in SPSS
    - Running a PCA with 2 components in SPSS
  - Common factor analysis
    - Principal axis factoring (2-factor PAF)
    - Maximum likelihood (2-factor ML)
- Rotation methods
  - Simple Structure
  - Orthogonal rotation (Varimax)
  - Oblique (Direct Oblimin)
- Generating factor scores

Introduction

- Motivating example: The SAQ
- Pearson correlation
- Partitioning the variance in factor analysis

Factors (latent) and Items (observed)



Observed variables

Latent variable

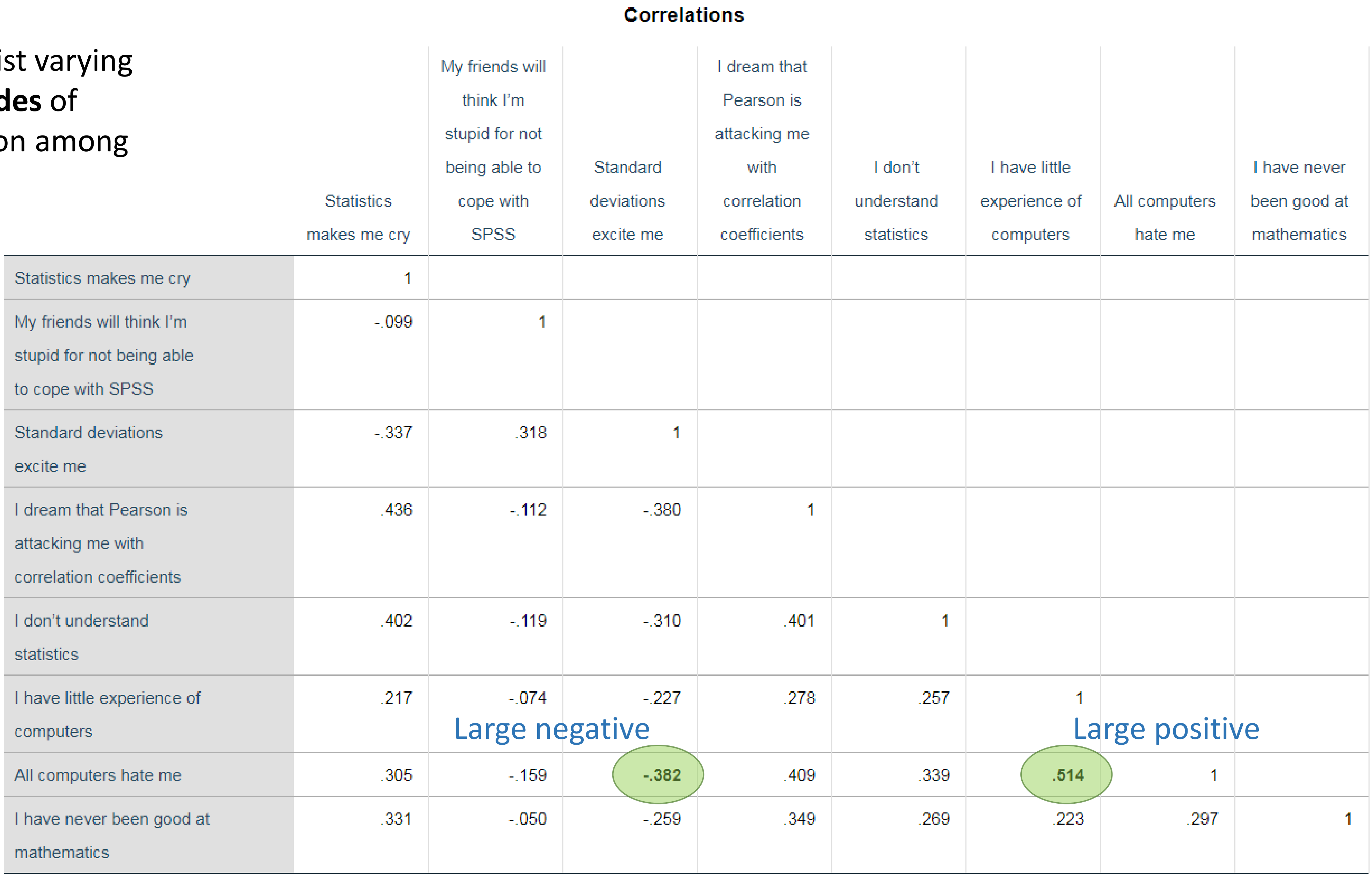
Assumption: the correlations among all observed variables can be explained by latent variable

SPSS Anxiety Questionnaire (SAQ-8)

1. **I dream that Pearson is attacking me with correlation coefficients**
2. **I don’t understand statistics**
3. **I have little experience with computers**
4. **All computers hate me**
5. **I have never been good at mathematics**
6. **My friends are better at statistics than me**
7. **Computers are useful only for playing games**
8. **I did badly at mathematics at school**

Pearson Correlation of the SAQ-8

There exist varying **magnitudes** of correlation among variables



Correlations

	Statistics makes me cry	My friends will think I'm stupid for not being able to cope with SPSS	Standard deviations excite me	I dream that Pearson is attacking me with correlation coefficients	I don't understand statistics	I have little experience of computers	All computers hate me	I have never been good at mathematics
Statistics makes me cry	1							
My friends will think I'm stupid for not being able to cope with SPSS	-.099	1						
Standard deviations excite me	-.337	.318	1					
I dream that Pearson is attacking me with correlation coefficients	.436	-.112	-.380	1				
I don't understand statistics	.402	-.119	-.310	.401	1			
I have little experience of computers	.217	-.074	-.227	.278	.257	1		
All computers hate me	.305	-.159	-.382	.409	.339	.514	1	
I have never been good at mathematics	.331	-.050	-.259	.349	.269	.223	.297	1

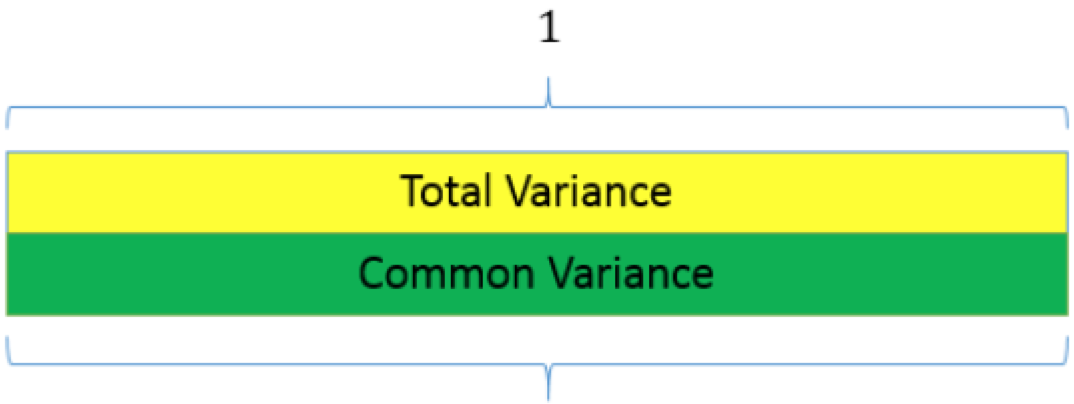
Large negative

Large positive

Partitioning the variance in factor analysis

- **Common variance**
  - variance that is shared among a set of *items*
  - **Communality** (h^2)
    - common variance that ranges between 0 and 1
- **Unique variance**
  - variance that's not common
  - **Specific variance**
    - variance that is specific to a particular item
    - Item 4 "All computers hate me" → anxiety about computers in addition to anxiety about SPSS
  - **Error variance**
    - anything unexplained by common or specific variance
    - e.g., a mother got a call from her babysitter that her two-year old son ate her favorite lipstick).

Variance Partitioning in PCA



1

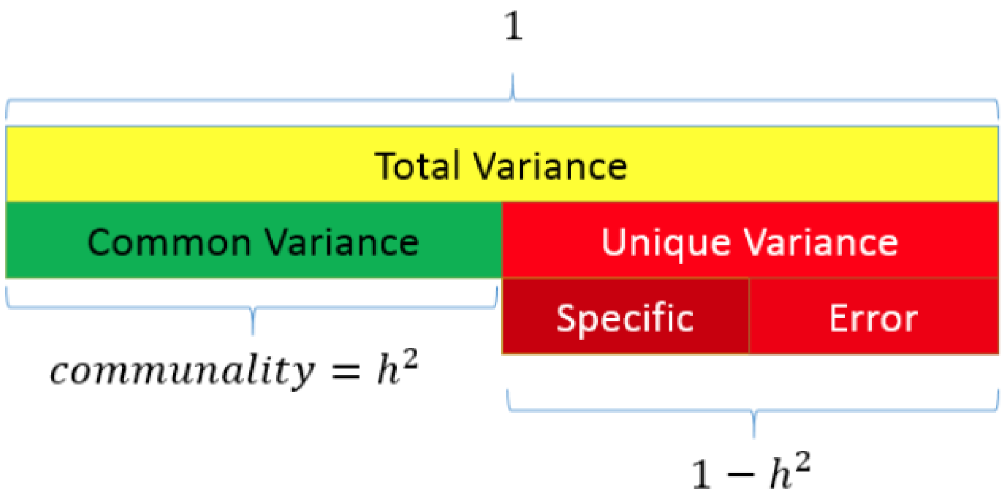
Total Variance

Common Variance

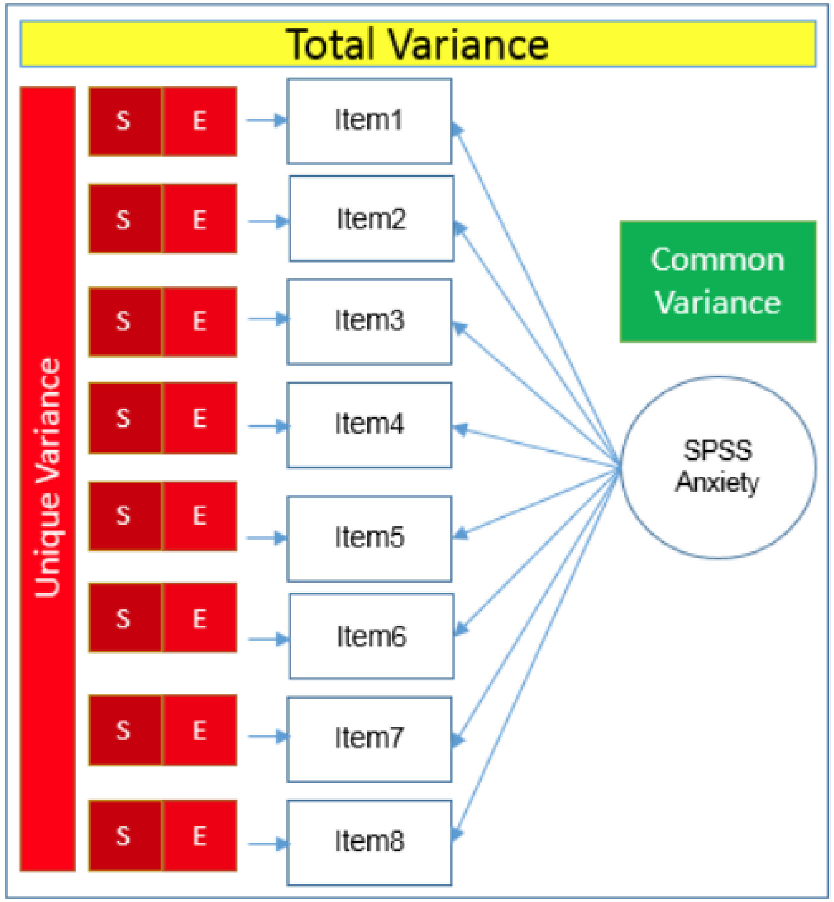
$$communality = h^2 = 1$$

In PCA, there is no unique variance. Common variance across a set of items makes up total variance.

Variance Partitioning in an EFA



Total variance is made up of common and unique variance



Common variance = Due to factor(s)

Unique variance = Due to items

Factor Extraction + Factor Rotation

- **Factor Extraction**
  - Type of model (e.g., PCA or EFA?)
  - Estimation method (e.g., Principal Axis Factoring or Maximum Likelihood?)
  - Number of factors or components to extract (e.g., 1 or 2?)

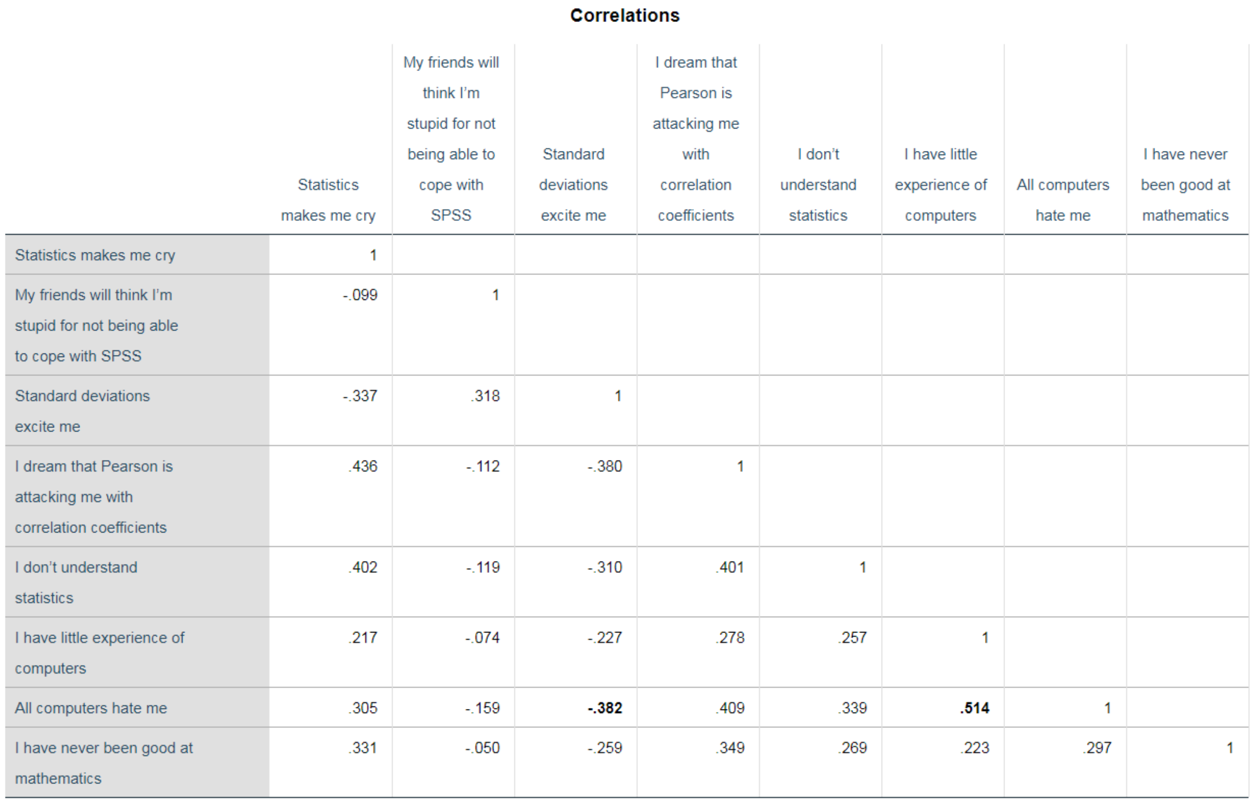
- **Factor Rotation**
  - Achieve simple structure
  - Orthogonal or oblique?

Extracting Factors

- Principal components analysis
    - PCA with 8 / 2 components
- Common factor analysis
    - Principal axis factoring (2-factor PAF)
    - Maximum likelihood (2-factor ML)

PCA

- **Principal Components Analysis (PCA)**
  - Goal: to replicate the correlation matrix using a set of components that are fewer in number than the original set of items



Correlations

	Statistics makes me cry	My friends will think I'm stupid for not being able to cope with SPSS	Standard deviations excite me	I dream that Pearson is attacking me with correlation coefficients	I don't understand statistics	I have little experience of computers	All computers hate me	I have never been good at mathematics
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All computers hate me	.305	-.159	**-.382**	.409	.339	**.514**	1	
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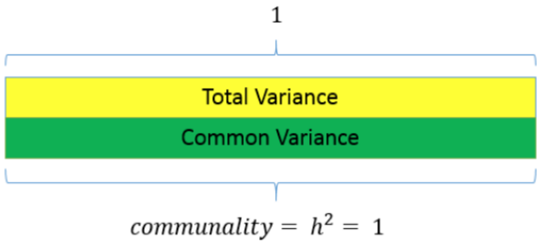
8 variables



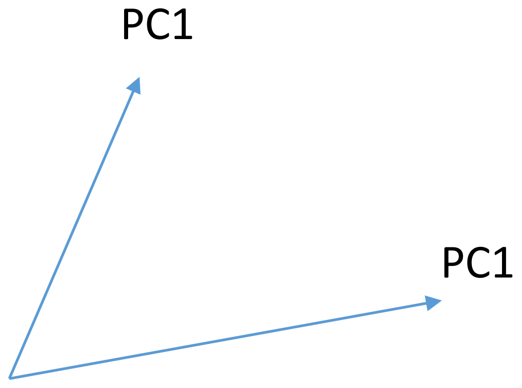
1

Total Variance

Common Variance

$communality = h^2 = 1$

Recall communality in PCA



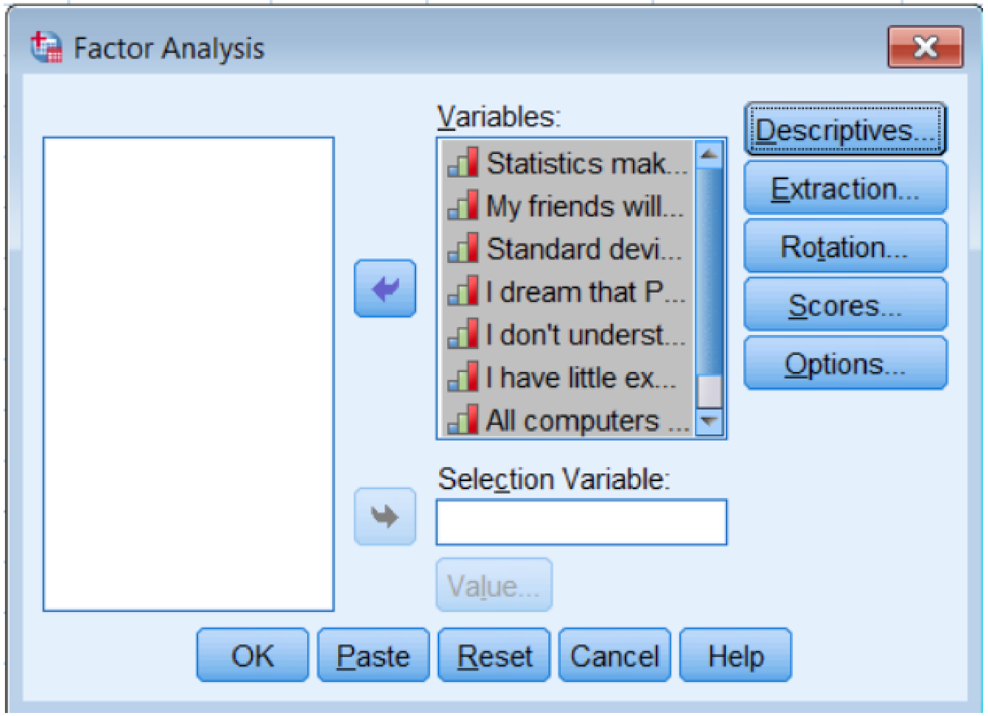
2 components

PCA: Eigenvalues and Eigenvectors

- **Eigenvalues**
  - Total variance explained by given principal component
  - Eigenvalues > 0, good
  - Negative eigenvalues → ill-conditioned
  - Eigenvalues close to zero → multicollinearity
- **Eigenvectors**
  - weight for each eigenvalue
  - eigenvector times the square root of the eigenvalue → component loadings
- **Component loadings**
  - correlation of each item with the principal component
  - *Eigenvalues* are the sum of squared component loadings across all items for each component

Running a PCA with 8 components

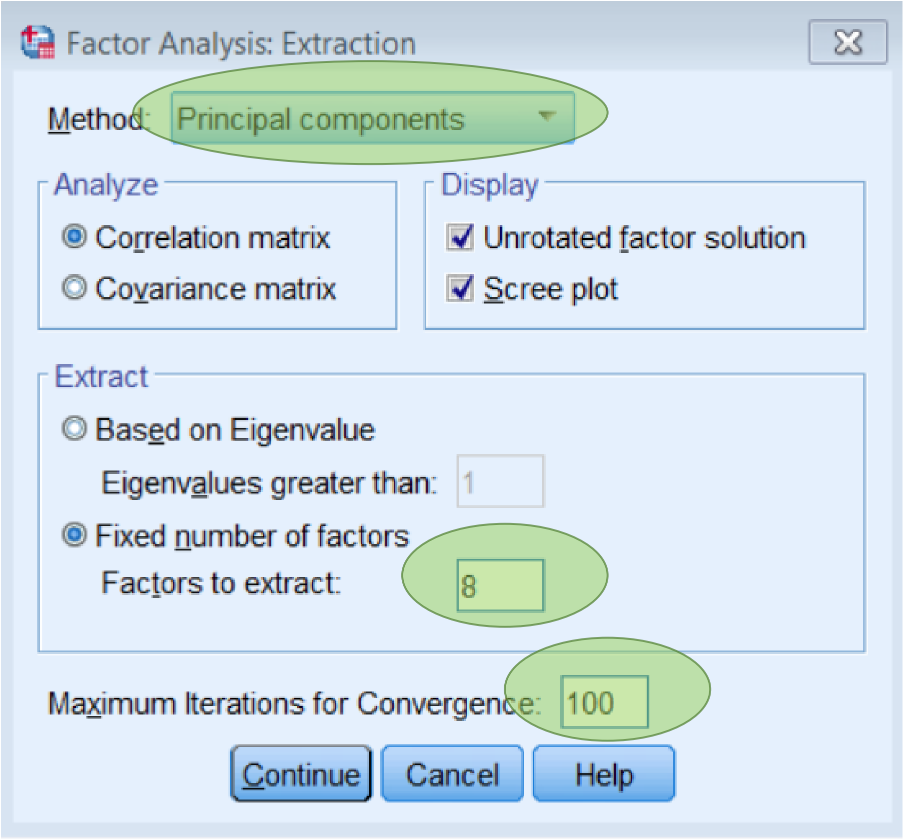
Analyze – Dimension Reduction – Factor



Factor Analysis

Variables:
- Statistics mak...
- My friends will...
- Standard devi...
- I dream that P...
- I don't underst...
- I have little ex...
- All computers ...

Selection Variable:



Factor Analysis: Extraction

Method: Principal components

Analyze
- ● Correlation matrix
- ○ Covariance matrix

Display
- ☑ Unrotated factor solution
- ☑ Scree plot

Extract
- ○ Based on Eigenvalue
  - Eigenvalues greater than: 1
- ● Fixed number of factors
  - Factors to extract: 8

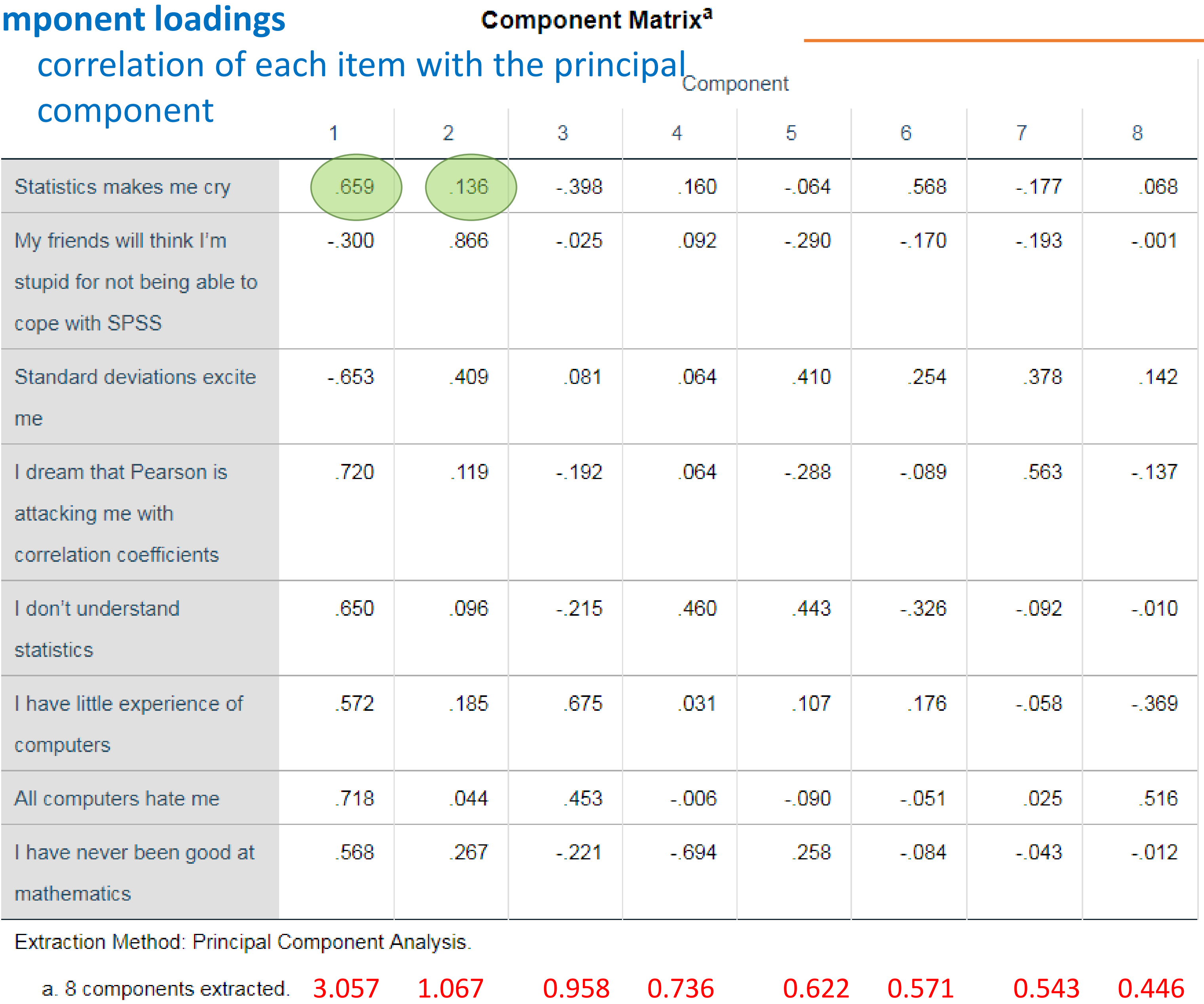
Maximum Iterations for Convergence: 100

Note: Factors are NOT the same as Components
8 components is NOT what you typically want to use

Component Matrix of the 8-component PCA

Component loadings

correlation of each item with the principal component



Component Matrix[a]

	Component								
	1	2	3	4	5	6	7	8	
Statistics makes me cry	.659	.136	-.398	.160	-.064	.568	-.177	.068	1
My friends will think I'm stupid for not being able to cope with SPSS	-.300	.866	-.025	.092	-.290	-.170	-.193	-.001	1
Standard deviations excite me	-.653	.409	.081	.064	.410	.254	.378	.142	1
I dream that Pearson is attacking me with correlation coefficients	.720	.119	-.192	.064	-.288	-.089	.563	-.137	1
I don't understand statistics	.650	.096	-.215	.460	.443	-.326	-.092	-.010	1
I have little experience of computers	.572	.185	.675	.031	.107	.176	-.058	-.369	1
All computers hate me	.718	.044	.453	-.006	-.090	-.051	.025	.516	1
I have never been good at mathematics	.568	.267	-.221	-.694	.258	-.084	-.043	-.012	1

Extraction Method: Principal Component Analysis.

a. 8 components extracted. 3.057 1.067 0.958 0.736 0.622 0.571 0.543 0.446

Sum of squared loadings across components is the communality

$0.659^2 = 0.434$

43.4% of the variance explained by first component (think R-square)

$0.136^2 = 0.018$

1.8% of the variance explained by second component

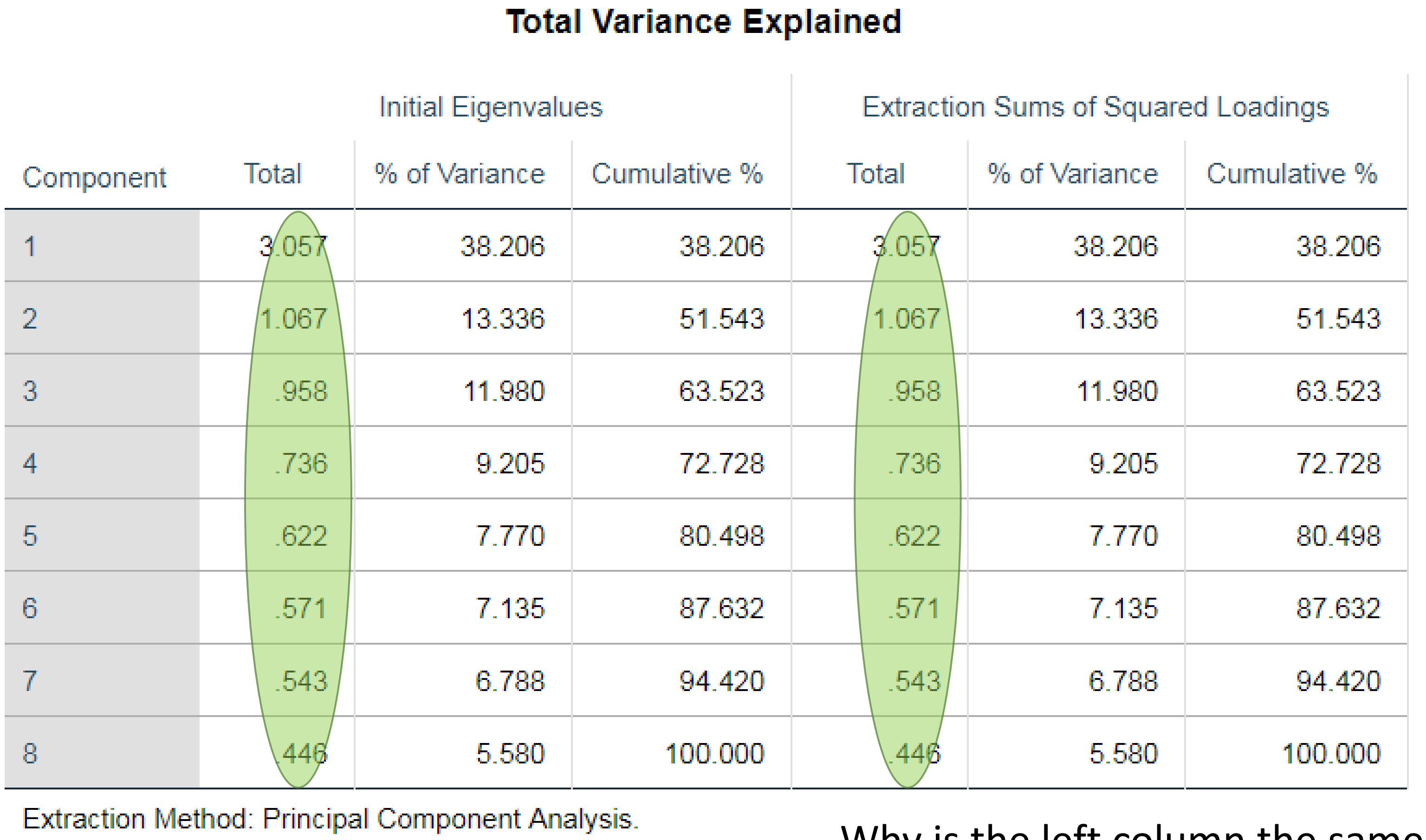
Q: why is it 1?

Excel demo

Sum squared loadings down each column (component) = eigenvalues

Total Variance Explained in the 8-component PCA

3.057 1.067 0.958 0.736 0.622 0.571 0.543 0.446



Total Variance Explained

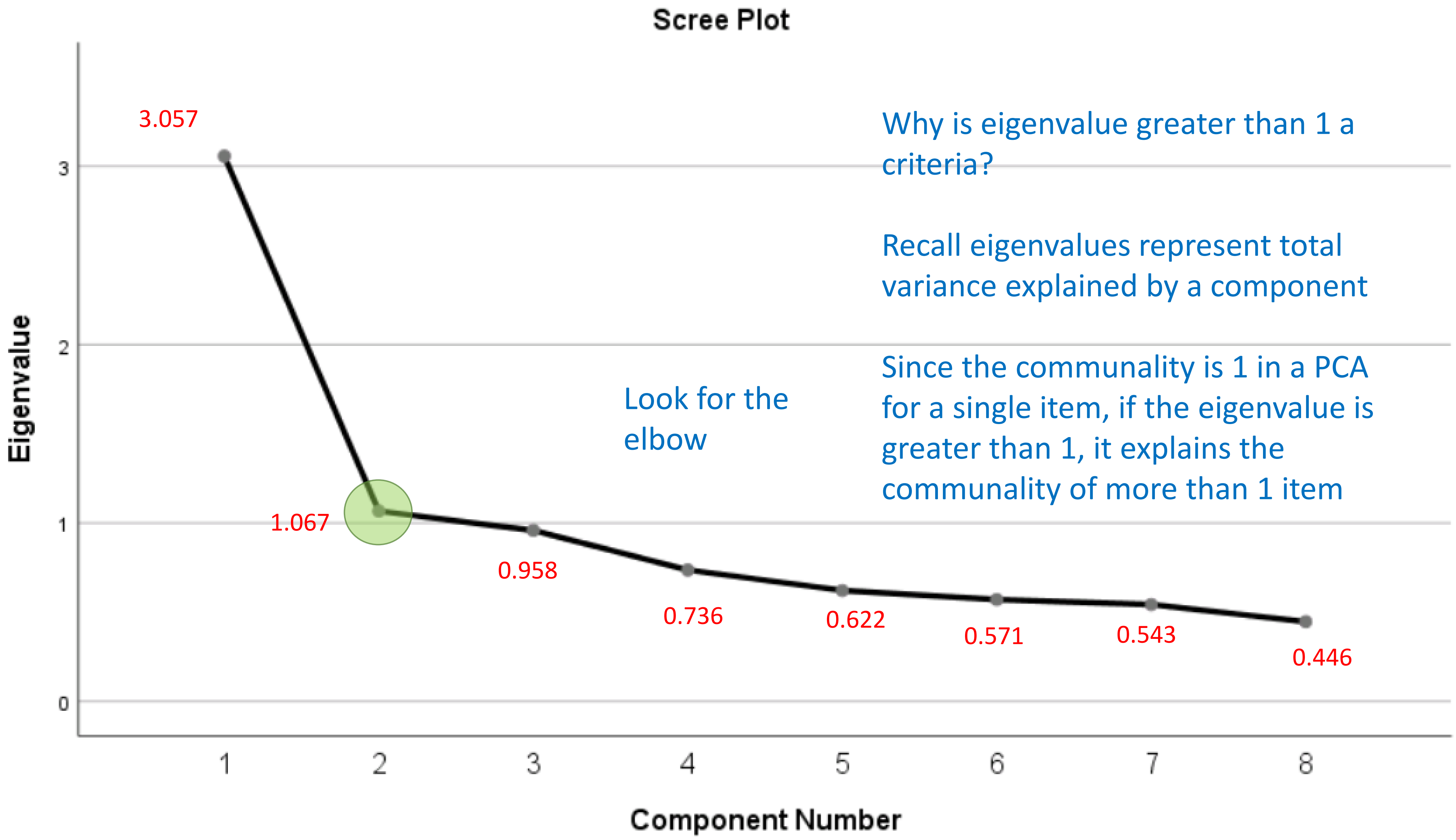
Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.057	38.206	38.206	3.057	38.206	38.206
2	1.067	13.336	51.543	1.067	13.336	51.543
3	.958	11.980	63.523	.958	11.980	63.523
4	.736	9.205	72.728	.736	9.205	72.728
5	.622	7.770	80.498	.622	7.770	80.498
6	.571	7.135	87.632	.571	7.135	87.632
7	.543	6.788	94.420	.543	6.788	94.420
8	.446	5.580	100.000	.446	5.580	100.000

Extraction Method: Principal Component Analysis.

Why is the left column the same as the right?

Look familiar? Extraction Sums of Squared Loadings = Eigenvalues

Choosing the number of components to extract



Look for the elbow

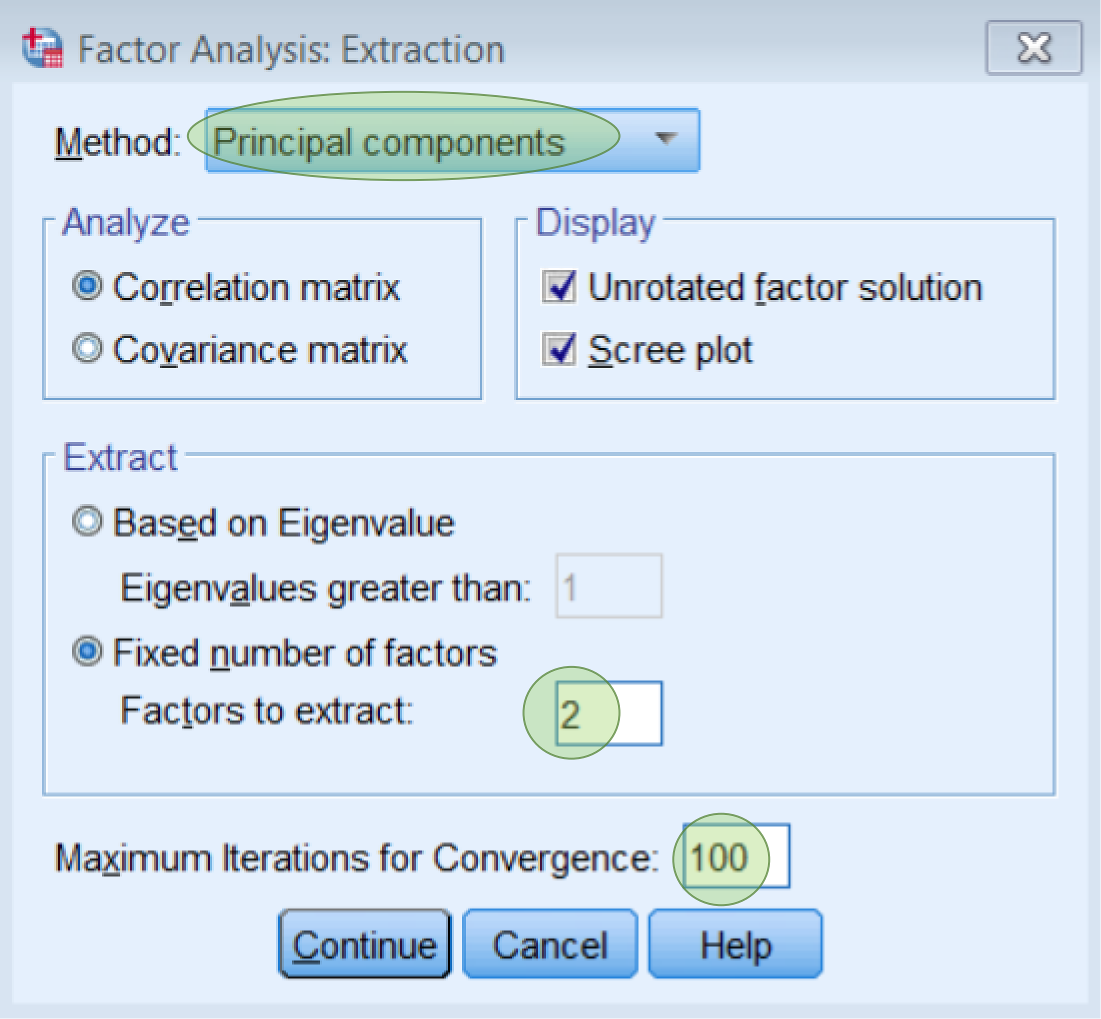
Why is eigenvalue greater than 1 a criteria?

Recall eigenvalues represent total variance explained by a component

Since the communality is 1 in a PCA for a single item, if the eigenvalue is greater than 1, it explains the communality of more than 1 item

Running a PCA with 2 components

Analyze – Dimension Reduction – Factor



Factor Analysis: Extraction

Method: Principal components

Analyze
- ● Correlation matrix
- ○ Covariance matrix

Display
- ☑ Unrotated factor solution
- ☑ Scree plot

Extract
- ○ Based on Eigenvalue
  - Eigenvalues greater than: 1
- ● Fixed number of factors
  - Factors to extract: 2

Maximum Iterations for Convergence: 100

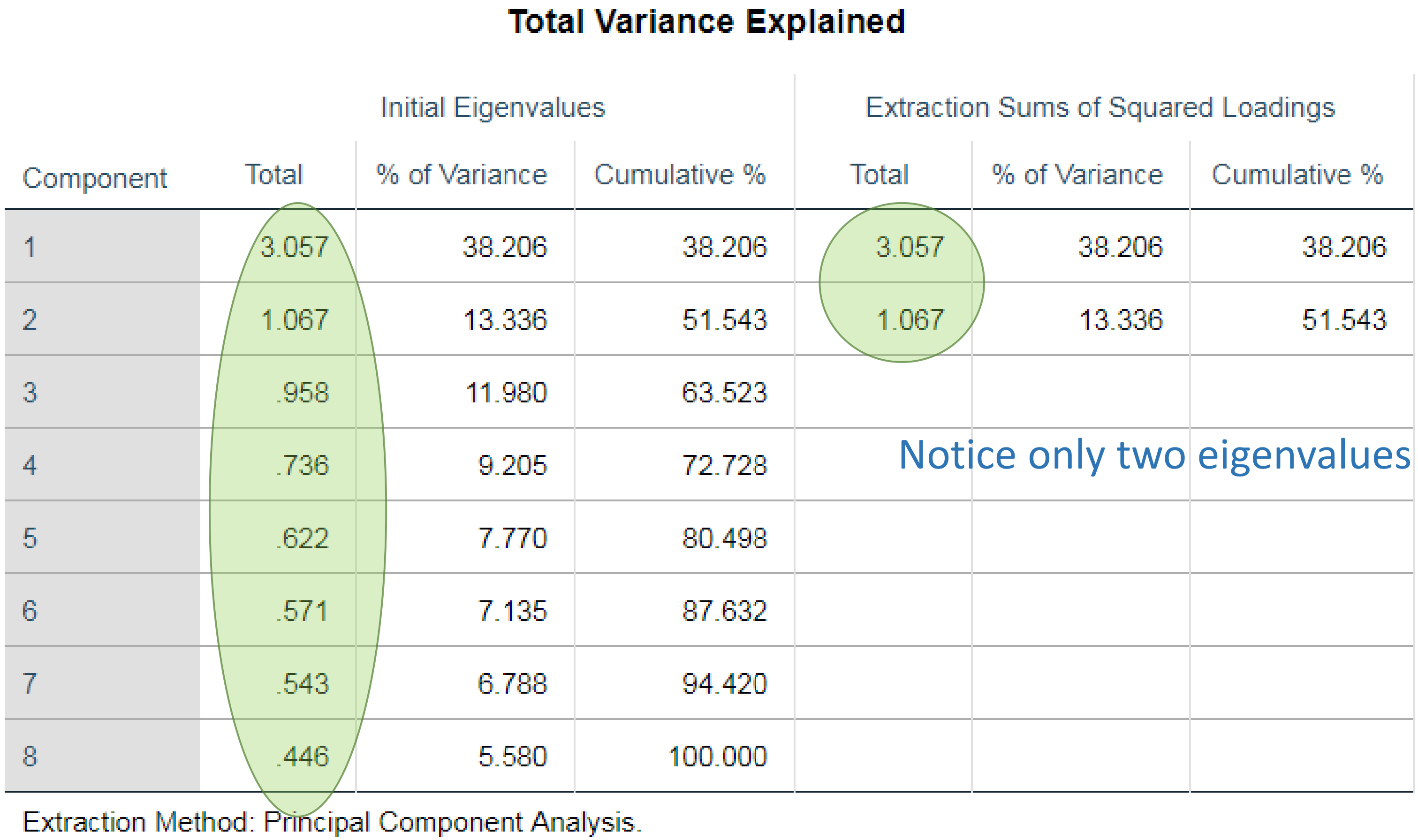
Goal of PCA is dimension reduction

This is more realistic than an 8-component solution

Output from 2-Component PCA

Recall these numbers from the 8-component solution

3.057 1.067 0.958 0.736 0.622 0.571 0.543 0.446



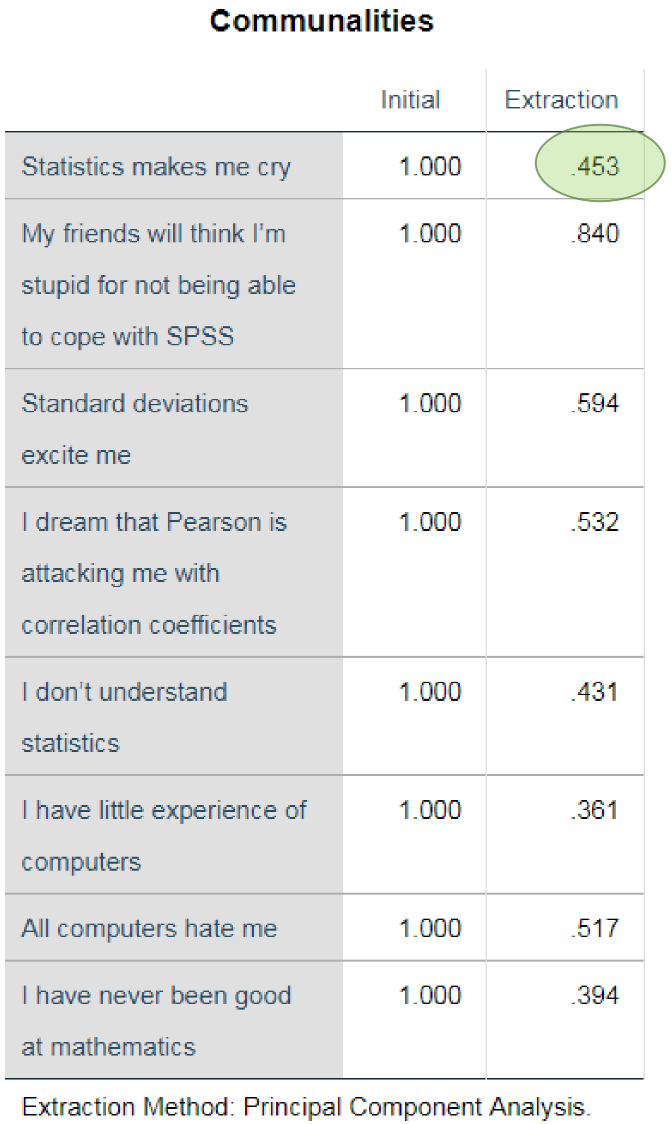
Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.057	38.206	38.206	3.057	38.206	38.206
2	1.067	13.336	51.543	1.067	13.336	51.543
3	.958	11.980	63.523			
4	.736	9.205	72.728			
5	.622	7.770	80.498			
6	.571	7.135	87.632			
7	.543	6.788	94.420			
8	.446	5.580	100.000			

Extraction Method: Principal Component Analysis.

Notice only two eigenvalues

Notice communalities not equal 1



Communalities

	Initial	Extraction
Statistics makes me cry	1.000	.453
My friends will think I'm stupid for not being able to cope with SPSS	1.000	.840
Standard deviations excite me	1.000	.594
I dream that Pearson is attacking me with correlation coefficients	1.000	.532
I don't understand statistics	1.000	.431
I have little experience of computers	1.000	.361
All computers hate me	1.000	.517
I have never been good at mathematics	1.000	.394

Extraction Method: Principal Component Analysis.

84.0% of the total variance in Item 2 is explained by Comp 1. How would you derive and interpret these communalities?

Quick Check 1

1. The elements of the Component Matrix are correlations of the item with each component. (Single Choice)

   Answer 1: T

   Answer 2: F

2. The sum of the squared eigenvalues is the proportion of variance under Total Variance Explained. (Single Choice)

   Answer 1: T

   Answer 2: F

3. The Component Matrix can be thought of as correlations and the Total Variance Explained table can be thought of as R-square. (Single Choice)

   Answer 1: T

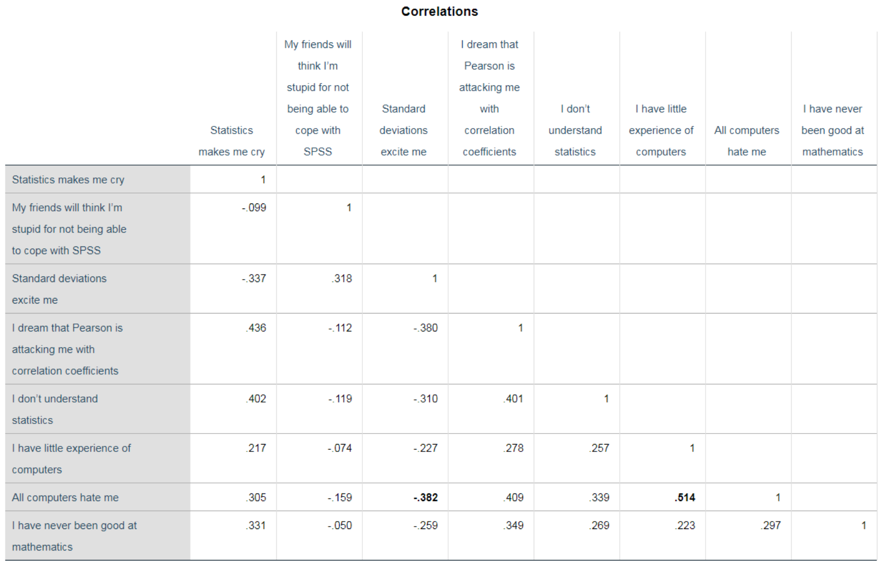
   Answer 2: F

Extracting Factors

- Principal components analysis
  - PCA with 8 / 2 components
- Common factor analysis
  - Principal axis factoring (2-factor PAF)
  - Maximum likelihood (2-factor ML)

Factor Analysis

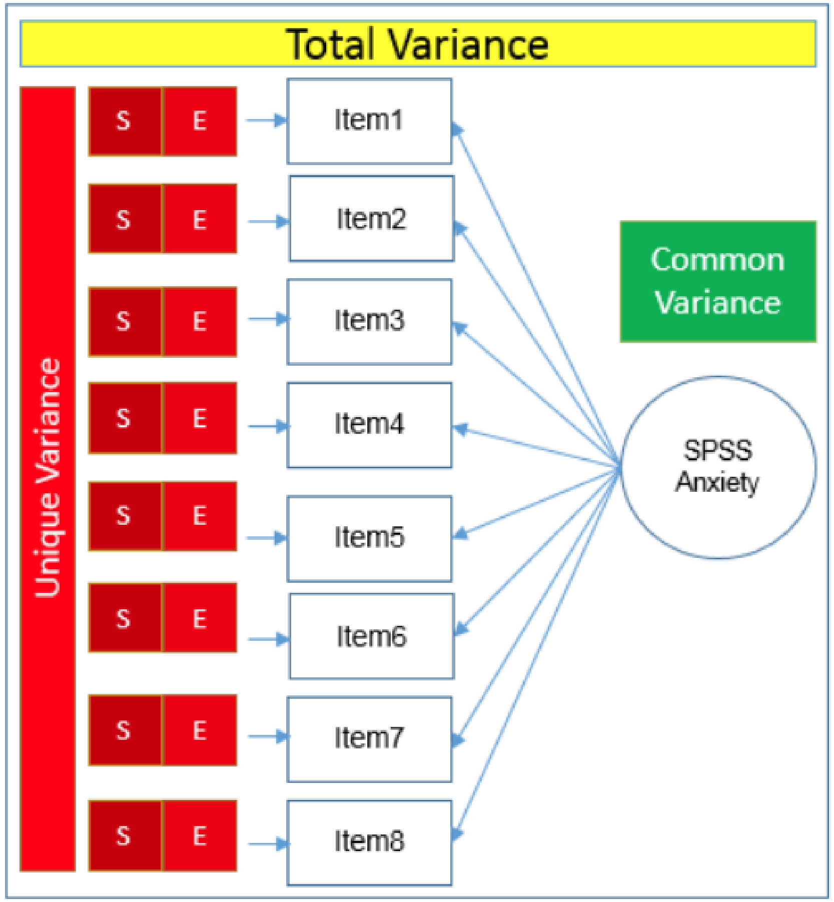
- **Factor Analysis (EFA)**
  - Goal: also to reduce dimensionality, BUT assume total variance can be divided into common and unique variance
    - Makes more sense to define a **construct** with measurement error



Correlations

	Statistics makes me cry	My friends will think I'm stupid for not being able to cope with SPSS	Standard deviations excite me	I dream that Pearson is attacking me with correlation coefficients	I don't understand statistics	I have little experience of computers	All computers hate me	I have never been good at mathematics
Statistics makes me cry	1							
My friends will think I'm stupid for not being able to cope with SPSS	-.099	1						
Standard deviations excite me	-.337	.318	1					
I dream that Pearson is attacking me with correlation coefficients	.436	-.112	-.380	1				
I don't understand statistics	.402	-.119	-.310	.401	1			
I have little experience of computers	.217	-.074	-.227	.278	.257	1		
All computers hate me	.305	-.159	**-.382**	.409	.339	**.514**	1	
I have never been good at mathematics	.331	-.050	-.259	.349	.269	.223	.297	1

8 variables

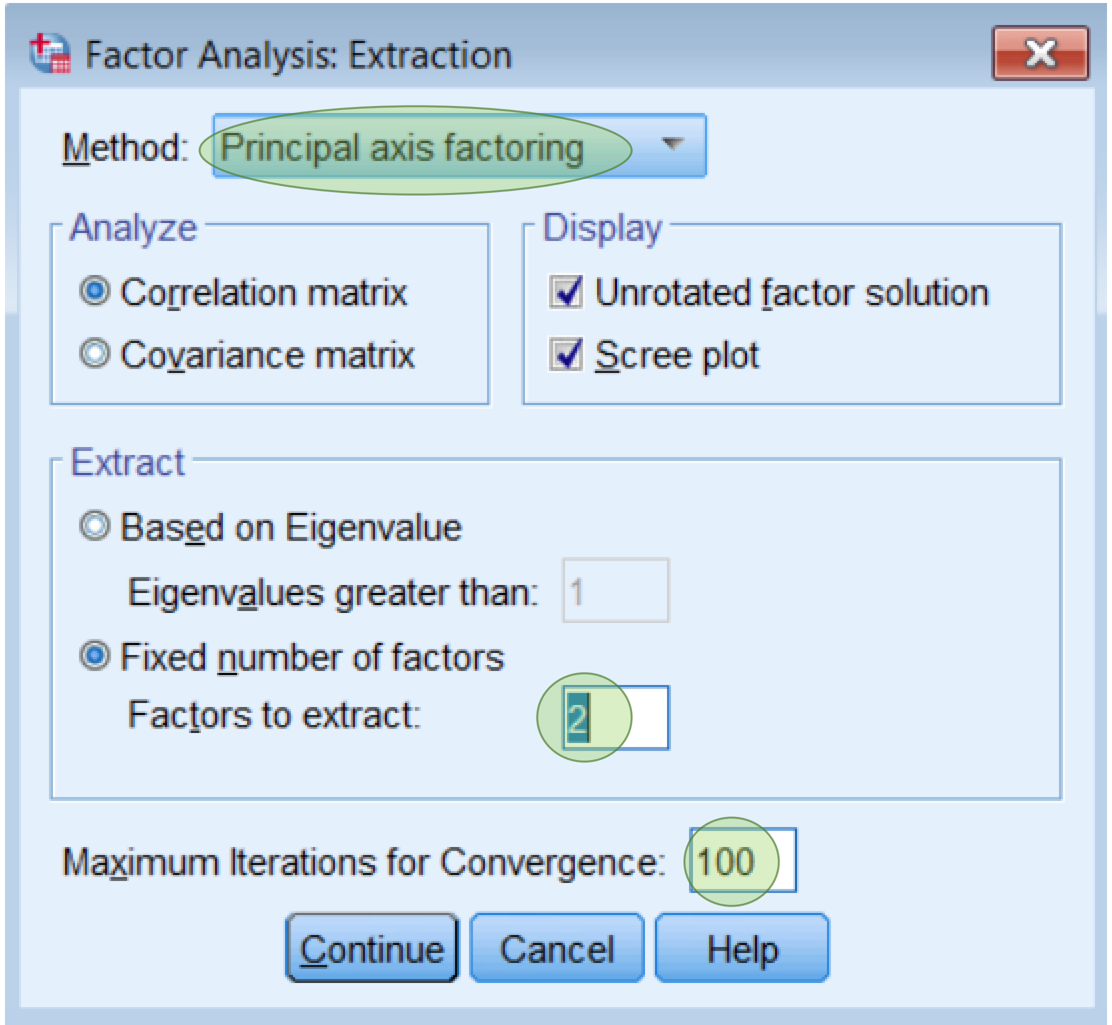


1 variable = factor

Common Factor Analysis with 2 factors (PAF)

Analyze – Dimension Reduction – Factor

Make note of the word *eigenvalue* it will come back to haunt us later



Factor Analysis: Extraction

Method: Principal axis factoring

Analyze
- ● Correlation matrix
- ○ Covariance matrix

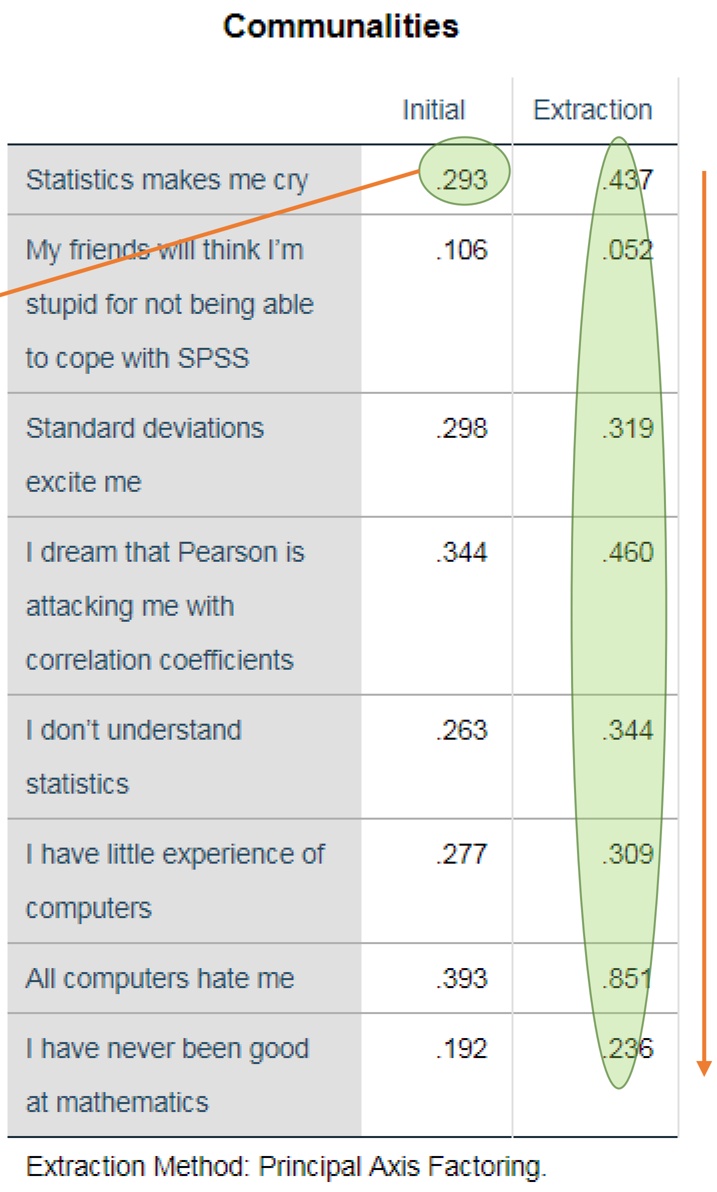
Display
- ☑ Unrotated factor solution
- ☑ Scree plot

Extract
- ○ Based on Eigenvalue
  - Eigenvalues greater than: 1
- ● Fixed number of factors
  - Factors to extract: 2

Maximum Iterations for Convergence: 100

SPSS does not change its menu to reflect changes in your analysis. You have to know the idiosyncrasies yourself.

Communalities of the 2-factor PAF



Communalities

	Initial	Extraction
Statistics makes me cry	.293	.437
My friends will think I'm stupid for not being able to cope with SPSS	.106	.052
Standard deviations excite me	.298	.319
I dream that Pearson is attacking me with correlation coefficients	.344	.460
I don't understand statistics	.263	.344
I have little experience of computers	.277	.309
All computers hate me	.393	.851
I have never been good at mathematics	.192	.236

Extraction Method: Principal Axis Factoring.

Initial communalities are the squared multiple correlation coefficients controlling for all other items in your model

Q: what was the initial communality for PCA?

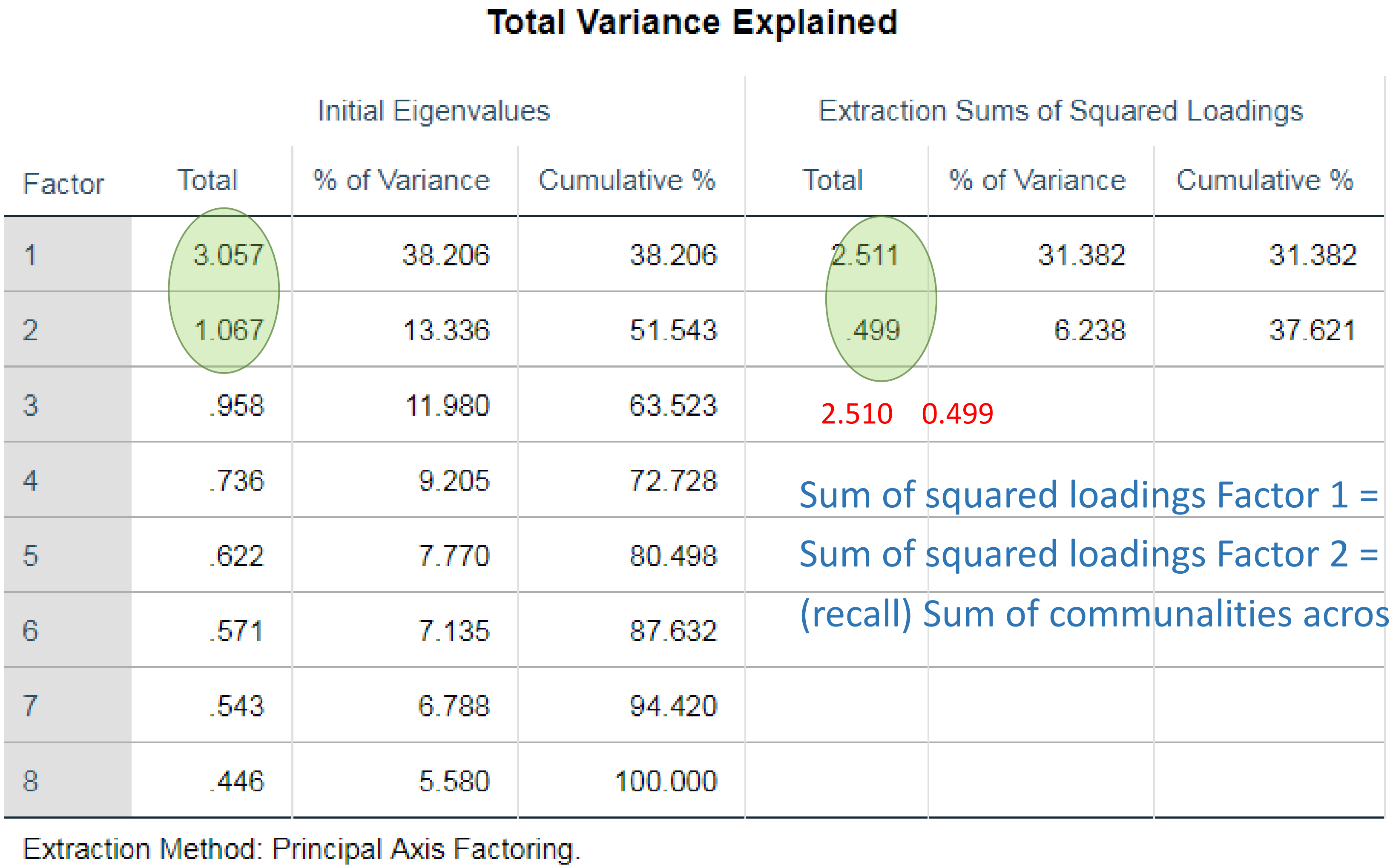
Sum of communalities across items = 3.01

Total Variance Explained (2-factor PAF)

Unlike the PCA model, the sum of the initial eigenvalues do not equal the sums of squared loadings

Sum eigenvalues = 4.124

The reason is because Eigenvalues are for PCA not for factor analysis! (SPSS idiosyncrasies)



Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.057	38.206	38.206	2.511	31.382	31.382
2	1.067	13.336	51.543	.499	6.238	37.621
3	.958	11.980	63.523			
4	.736	9.205	72.728			
5	.622	7.770	80.498			
6	.571	7.135	87.632			
7	.543	6.788	94.420			
8	.446	5.580	100.000			

Extraction Method: Principal Axis Factoring.

2.510 0.499

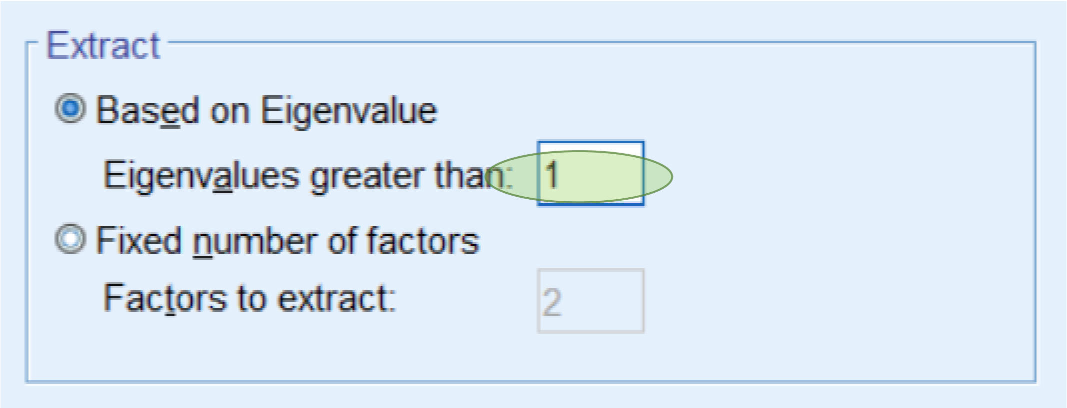
Sum of squared loadings Factor 1 = 2.51

Sum of squared loadings Factor 2 = 0.499

(recall) Sum of communalities across items = 3.01

Eigenvalues do not belong in EFA!

Analyze – Dimension Reduction – Factor



Extract

● Based on Eigenvalue

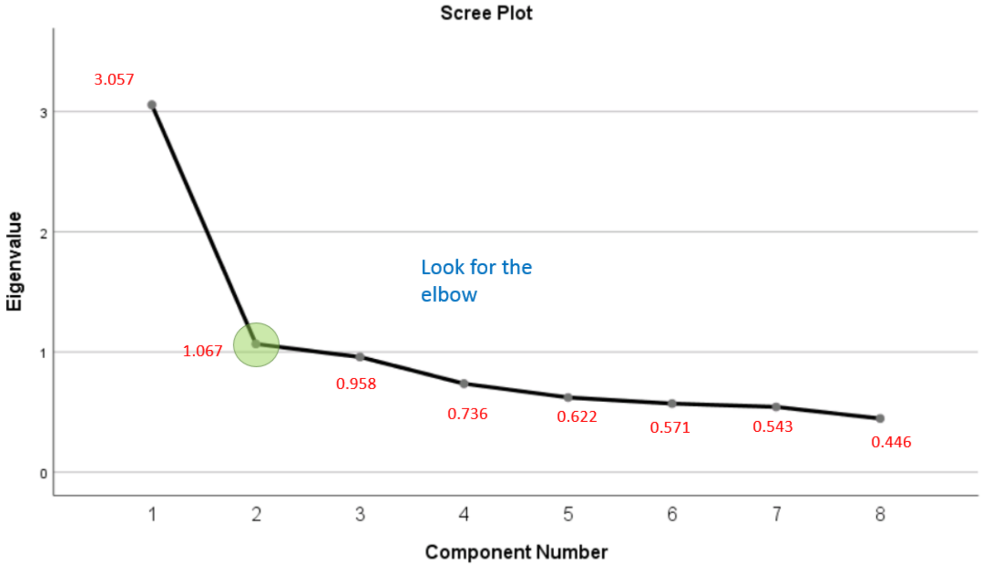
Eigenvalues greater than: 1

○ Fixed number of factors

Factors to extract: 2

Caution!

Eigenvalues are only for PCA, yet SPSS uses the eigenvalue criteria for EFA



When you look at the scree plot in SPSS, you are making a conscious decision to use the PCA solution as a proxy for your EFA

Quick Check 2

1. In the Total Variance Explained table, the percent of variance in the Initial column equals the Extraction column when... (Single Choice)

Answer 1: You run a factor analysis.

Answer 2: There is no error variance.

Answer 3: There is no unique variance.

2. In SPSS when you use the Principal Axis Factor method the scree plot uses the final factor analysis solution to plot the eigenvalues. (Single Choice)

Answer 1: T

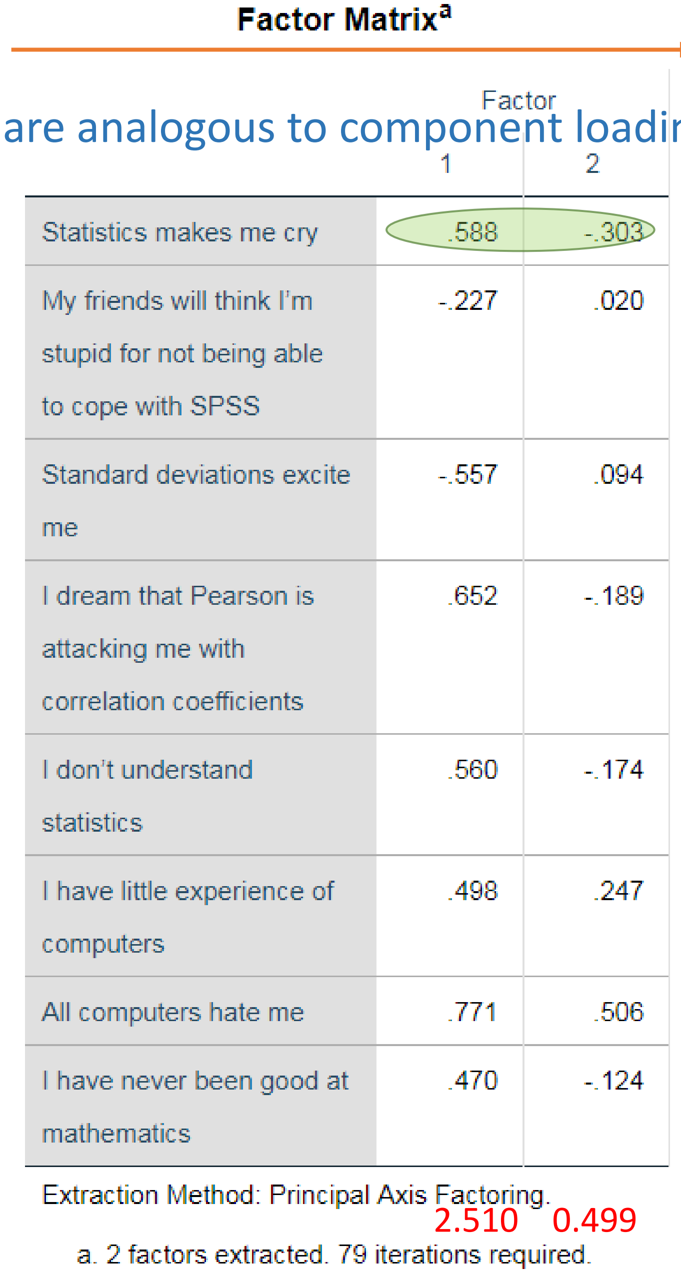
Answer 2: F

Factor Matrix (2-factor PAF)

These are analogous to component loadings in PCA

Squaring the loadings and summing up gives you either the *Communality* or the *Extraction Sums of Squared Loadings*

Summing down the communalities or across the eigenvalues gives you **total variance explained (3.01)**



Factor Matrix[a]

	Factor 1	Factor 2
Statistics makes me cry	.588	-.303
My friends will think I'm stupid for not being able to cope with SPSS	-.227	.020
Standard deviations excite me	-.557	.094
I dream that Pearson is attacking me with correlation coefficients	.652	-.189
I don't understand statistics	.560	-.174
I have little experience of computers	.498	.247
All computers hate me	.771	.506
I have never been good at mathematics	.470	-.124

Extraction Method: Principal Axis Factoring.

2.510 0.499

a. 2 factors extracted. 79 iterations required.

0.438

0.052

0.319

0.461

0.344

0.309

0.850

0.236

3.01

Sum of squared loadings across factors is the communality

$0.588^2 = 0.346$

34.5% of the variance in Item 1 explained by first factor

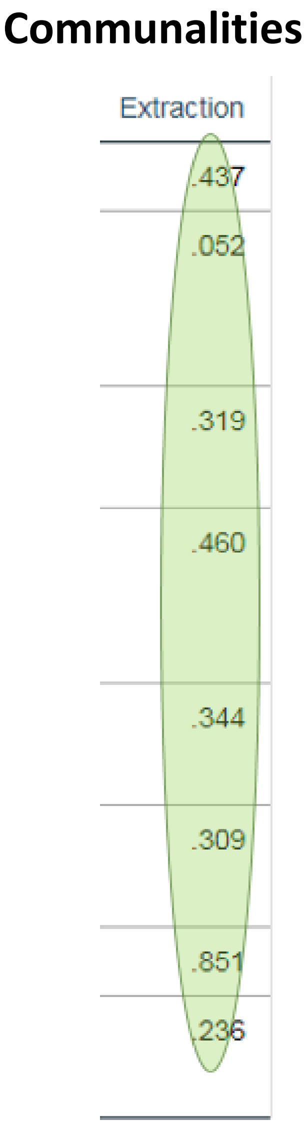
$(-0.303)^2 = 0.091$

9.1% of the variance in Item 1 explained by second factor

$0.345 + 0.091 = 0.437$

43.7% of the variance in Item 1 explained by both factors = COMMUNALITY!

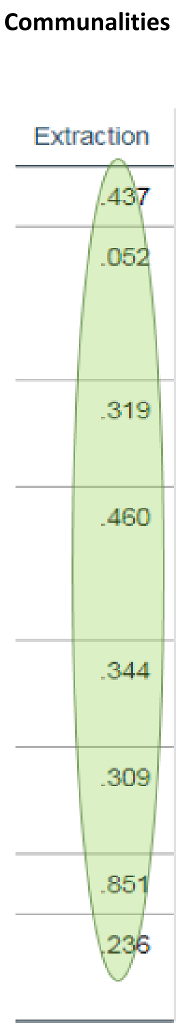
Sum squared loadings down each column = Extraction Sums of Square Loadings (not eigenvalues)



Communalities

Extraction
.437
.052
.319
.460
.344
.309
.851
.236

Path Diagram



Communalities

Extraction
.437
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.851
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Factor Matrix[a]

	Factor	
	1	2
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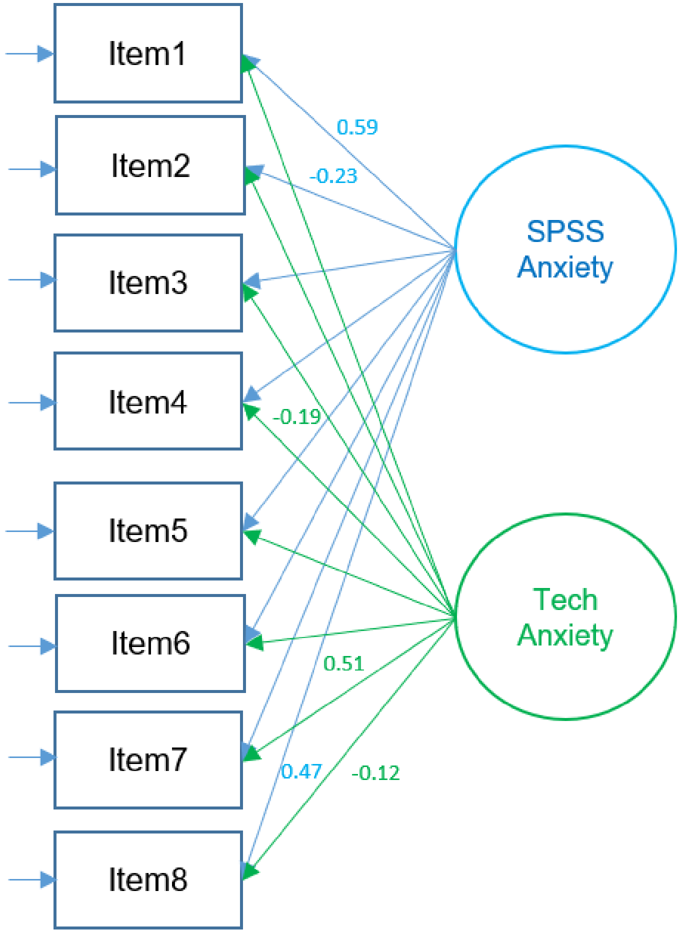
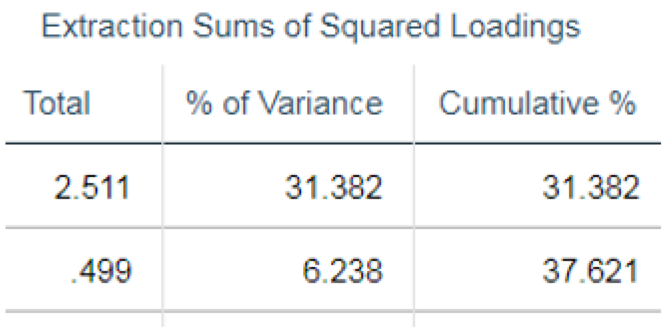
Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 79 iterations required.



Note: only selected loadings shown



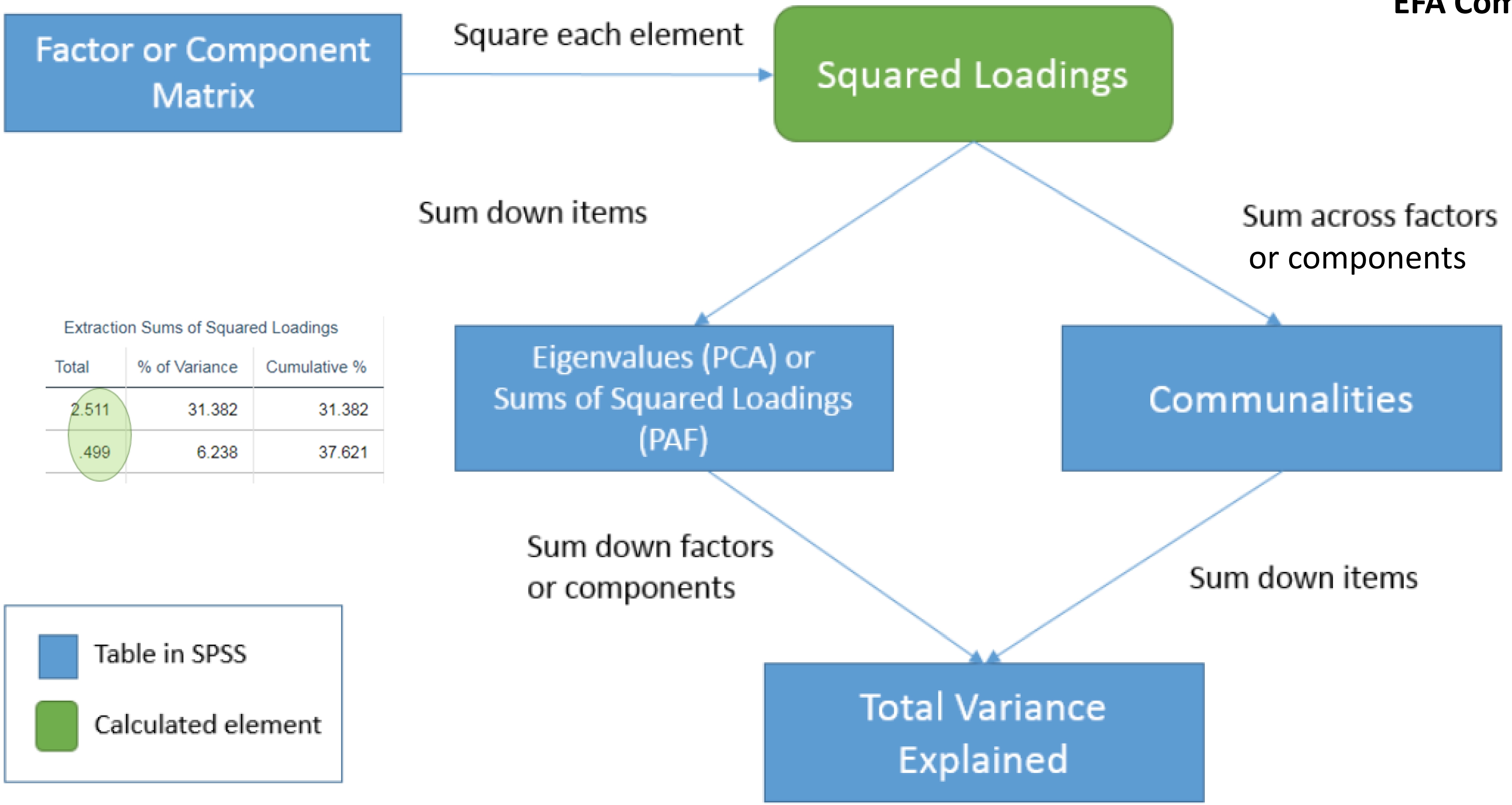
Extraction Sums of Squared Loadings

Total	% of Variance	Cumulative %
2.511	31.382	31.382
.499	6.238	37.621

Caution when interpreting unrotated loadings. Most of total variance explained by first factor.

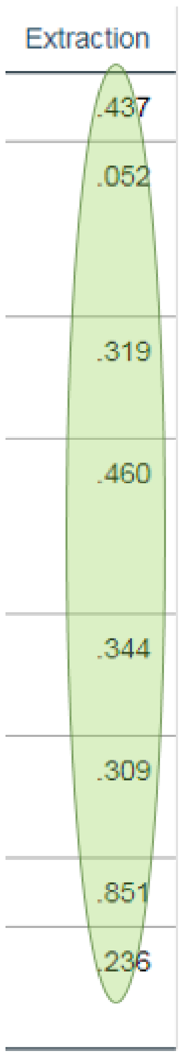
Which item has the least total variance explained by both factors?

The relationship between the three tables



Extraction Sums of Squared Loadings		
Total	% of Variance	Cumulative %
2.511	31.382	31.382
.499	6.238	37.621

EFA Communalities



Extraction
.437
.052
.319
.460
.344
.309
.851
.236

PCA **8** **3.01** EFA

Excel demo

Quick Check 3

1. The eigenvalue represents the communality for each item. (Single Choice)

   Answer 1: T

   Answer 2: F

2. For a single component, the sum of squared component loadings across all items represents the eigenvalue for that component. (Single Choice)

   Answer 1: T

   Answer 2: F

3. The sum of eigenvalues for all the components is the total variance. (Single Choice)

   Answer 1: T

   Answer 2: F

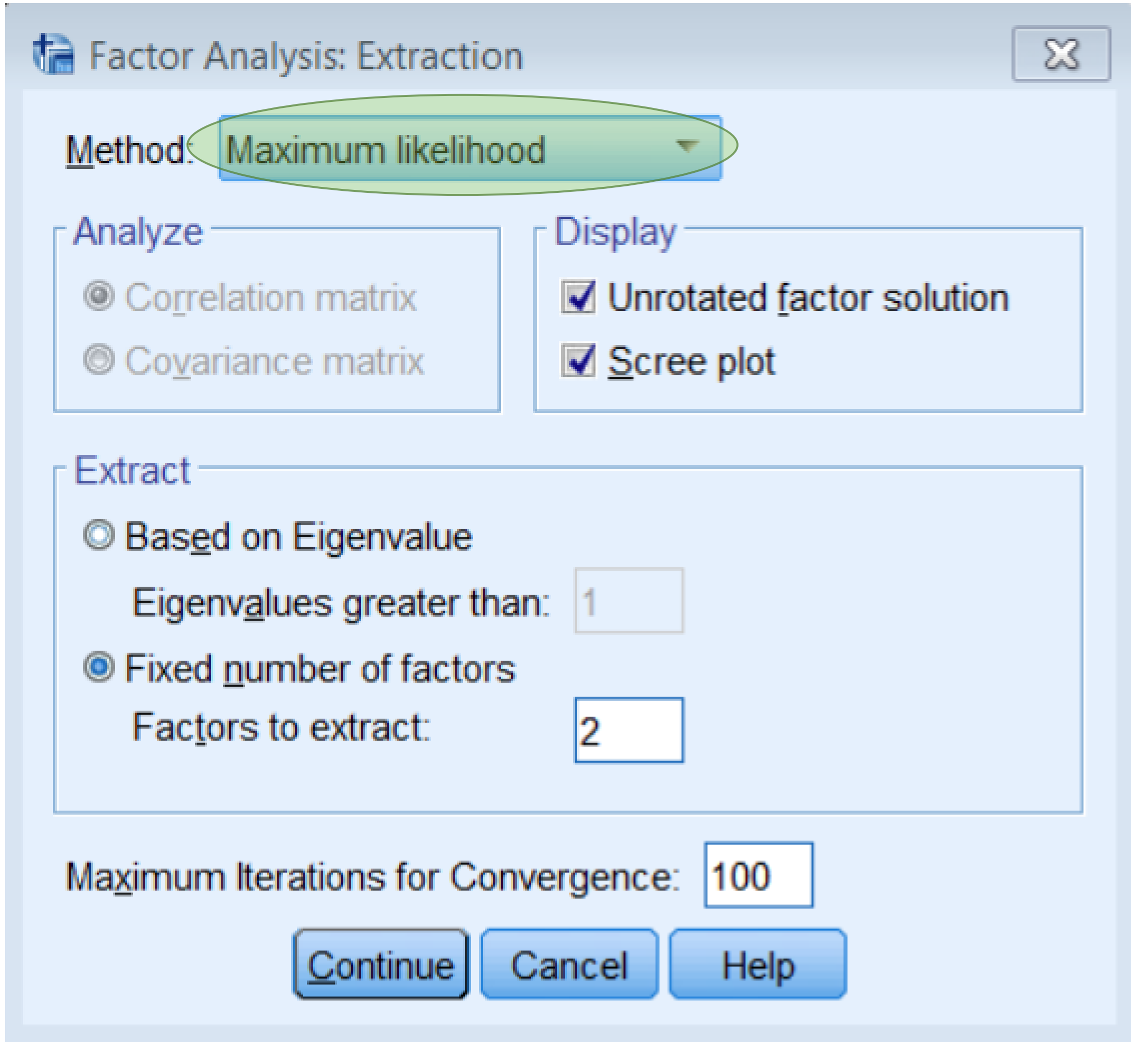
4. The sum of the communalities down the components is equal to the sum of eigenvalues down the items. (Single Choice)

   Answer 1: T

   Answer 2: F

Maximum Likelihood Estimation (2-factor ML)

Analyze – Dimension Reduction – Factor



Factor Analysis: Extraction

Method: Maximum likelihood

Analyze

● Correlation matrix

○ Covariance matrix

Display

☑ Unrotated factor solution

☑ Scree plot

Extract

○ Based on Eigenvalue

Eigenvalues greater than: 1

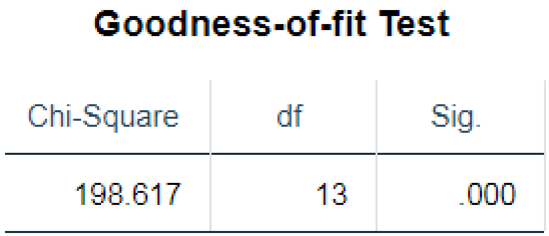
● Fixed number of factors

Factors to extract: 2

Maximum Iterations for Convergence: 100

New output

A significant chi-square means you *reject* the current hypothesized model



Goodness-of-fit Test

Chi-Square	df	Sig.
198.617	13	.000

This is telling us we reject the two-factor model

Chi-square Comparison Table

Chi-square and degrees of freedom goes down

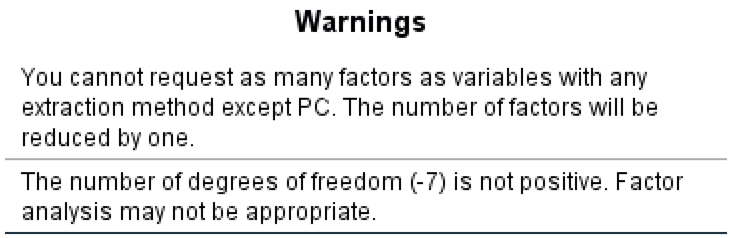
The three factor model is preferred from chi-square

Number of Factors	Chi-square	Df	p-value	Iterations needed
1	553.08	20	<0.05	4
2	198.62	13	< 0.05	39
3	13.81	7	0.055	57
4	1.386	2	0.5	168
5	NS	-2	NS	NS
6	NS	-5	NS	NS
7	NS	-7	NS	NS
8	N/A	N/A	N/A	N/A

Want NON-significant chi-square

Iterations needed goes up

An eight factor model is not possible in SPSS



Warnings

You cannot request as many factors as variables with any extraction method except PC. The number of factors will be reduced by one.

The number of degrees of freedom (-7) is not positive. Factor analysis may not be appropriate.

Quick Check 4

1. Since they are both factor analysis methods, Principal Axis Factoring and the Maximum Likelihood method will result in the same Factor Matrix. (Single Choice)

Answer 1: T

Answer 2: F

2. In SPSS, both Principal Axis Factoring and Maximum Likelihood methods give chi-square goodness of fit tests. (Single Choice)

Answer 1: T

Answer 2: F

3. When looking at the Goodness-of-fit Test table, a p-value less than 0.05 means the model is a good fitting model. (Single Choice)

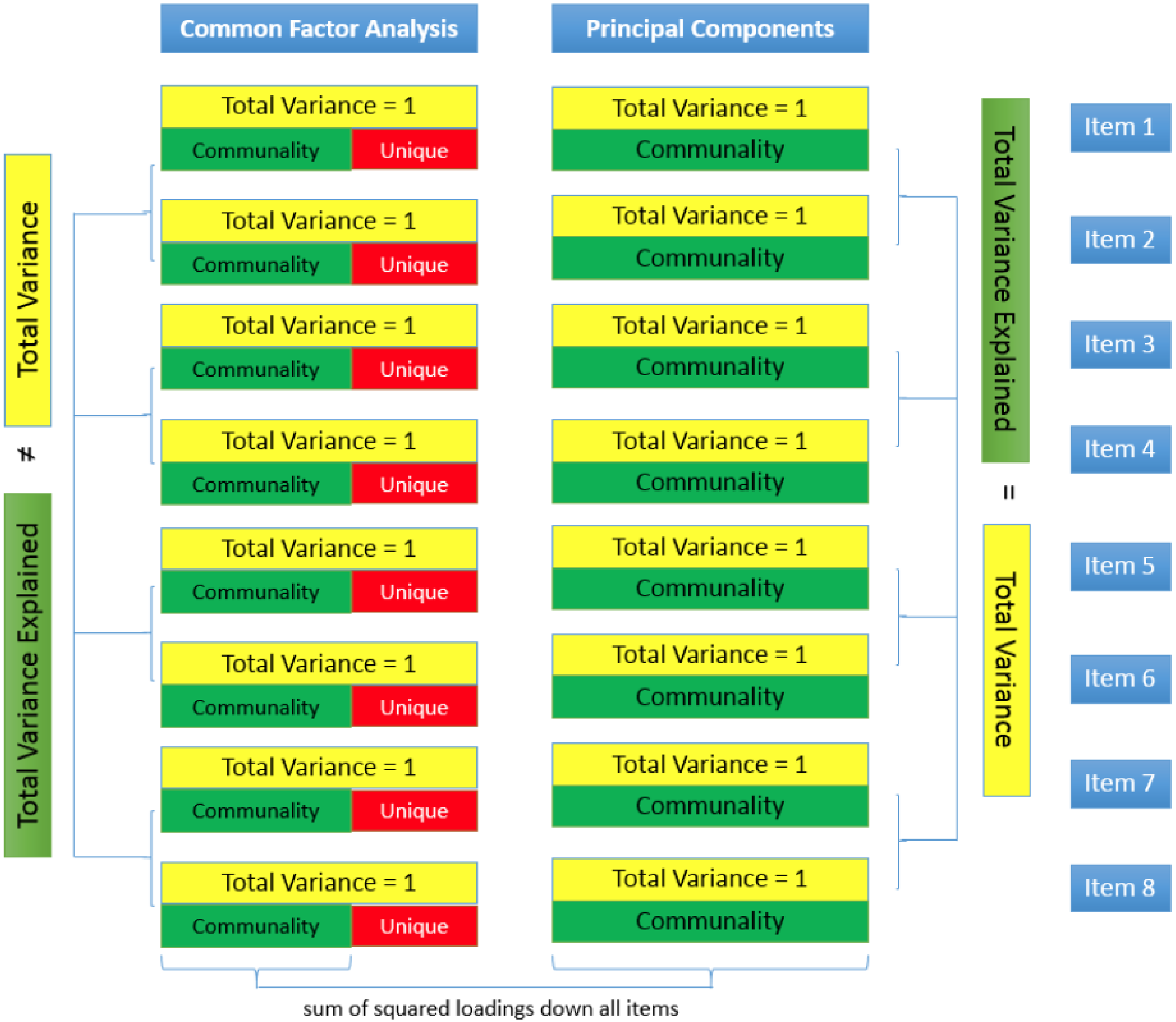
Answer 1: T

Answer 2: F

Comparing EFA with PCA

EFA: Total Variance Explained = Total Communality Explained NOT Total Variance

For both models, communality is the total proportion of variance due to all factors or components in the model



Common Factor Analysis

Total Variance

≠

Total Variance Explained

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Total Variance = 1
Communality | Unique

Principal Components

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance = 1
Communality

Total Variance Explained

=

Total Variance

Item 1
Item 2
Item 3
Item 4
Item 5
Item 6
Item 7
Item 8

sum of squared loadings down all items

PCA: Total Variance Explained = Total Variance

Communalities are item specific

Quick Check 5 (optional)

1. In Principal Axis Factoring, the elements of the Factor Matrix represent correlations of each item with a factor. (Single Choice)

Answer 1: T

Answer 2: F

2. In common factor analysis, the Sums of Squared loadings is the same as the eigenvalue. (Single Choice)

Answer 1: T

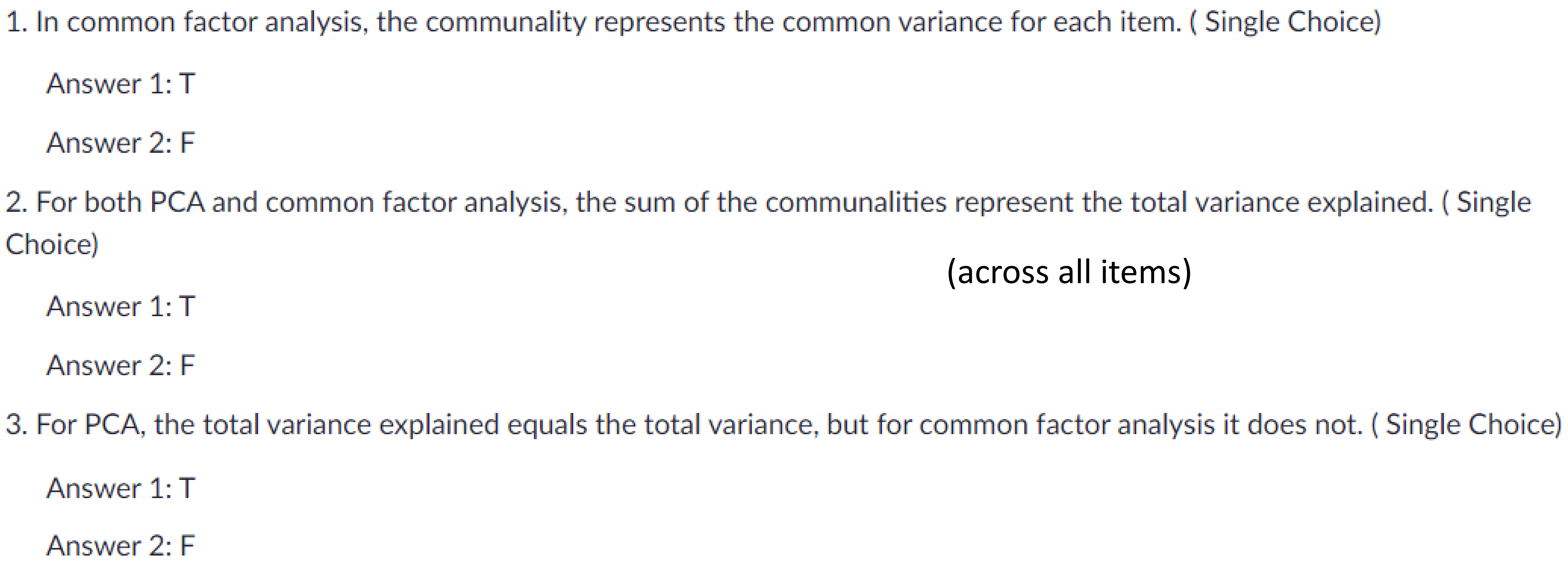
Answer 2: F

3. In EFA, the Sum of Squared Loadings across all items in the Factor Matrix represents the communality. (Single Choice)

Answer 1: T

Answer 2: F

Quick Check 6



1. In common factor analysis, the communality represents the common variance for each item. (Single Choice)

Answer 1: T

Answer 2: F

2. For both PCA and common factor analysis, the sum of the communalities represent the total variance explained. (Single Choice)

Answer 1: T

Answer 2: F

3. For PCA, the total variance explained equals the total variance, but for common factor analysis it does not. (Single Choice)

Answer 1: T

Answer 2: F

(across all items)

Rotation Methods

- Simple Structure
- Orthogonal rotation (Varimax)
- Oblique (Direct Oblimin)

Simple structure

1. Each item has high loadings on one factor only
2. Each factor has high loadings for only some of the items.

Pedhazur and Schemlkin (1991)

Item	Factor 1	Factor 2	Factor 3
1	0.8	0	0
2	0.8	0	0
3	0.8	0	0
4	0	0.8	0
5	0	0.8	0
6	0	0.8	0
7	0	0	0.8
8	0	0	0.8

The goal of rotation is to achieve simple structure

NOT simple structure

1. Most items have high loadings on *more* than one factor
2. Factor 3 has high loadings on 5/8 items

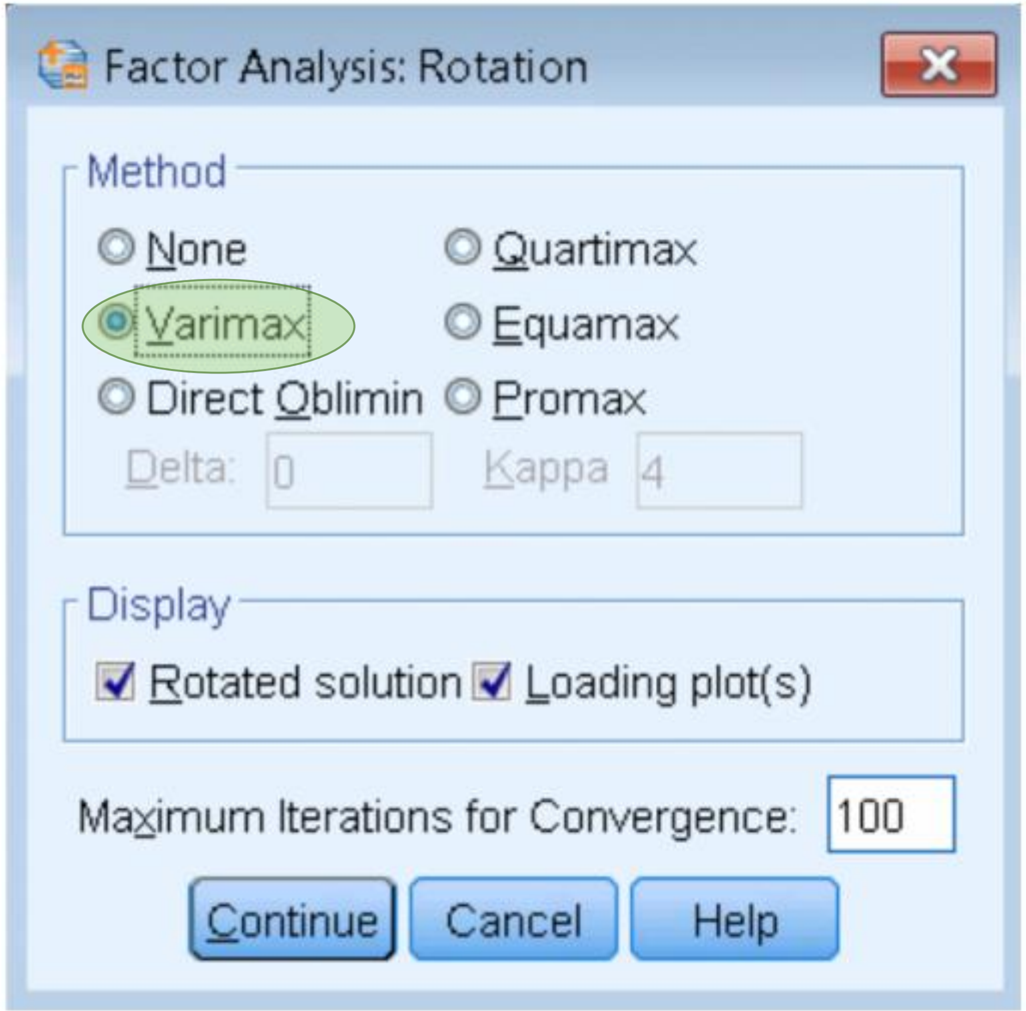
Item	Factor 1	Factor 2	Factor 3
1	0.8	0	0.8
2	0.8	0	0.8
3	0.8	0	0
4	0.8	0	0
5	0	0.8	0.8
6	0	0.8	0.8
7	0	0.8	0.8
8	0	0.8	0

Running a 2-factor solution (PAF Varimax rotation)

Without rotation, first factor is the most general factor onto which most items load and explains the largest amount of variance

Varimax:
orthogonal rotation

maximizes variances of the loadings within the factors while maximizing differences between high and low loadings on a particular factor



Factor Analysis: Rotation

Method

○ None ○ Quartimax

● Varimax ○ Equamax

○ Direct Oblimin ○ Promax

Delta: 0 Kappa 4

Display

☑ Rotated solution ☑ Loading plot(s)

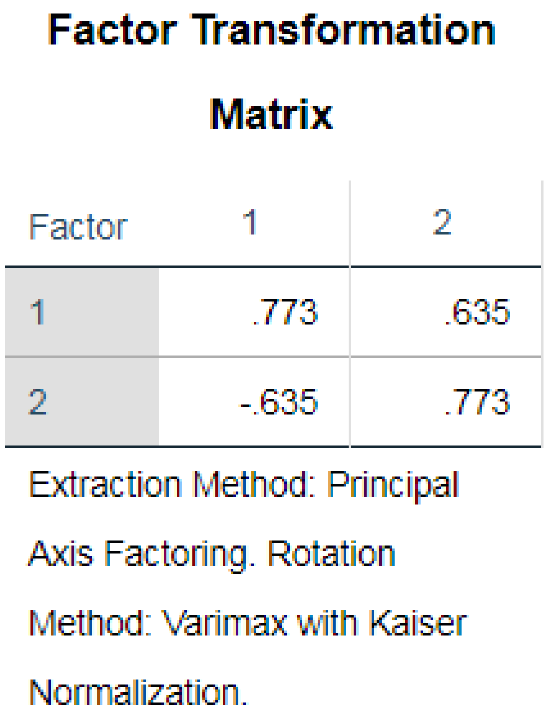
Maximum Iterations for Convergence: 100

Orthogonal means the factors are uncorrelated

Factor Transformation Matrix

Factor Matrix → Factor Transformation Matrix → Rotated Factor Matrix

The factor transformation matrix turns the regular factor matrix into the rotated factor matrix



Factor Transformation Matrix

Factor	1	2
1	.773	.635
2	-.635	.773

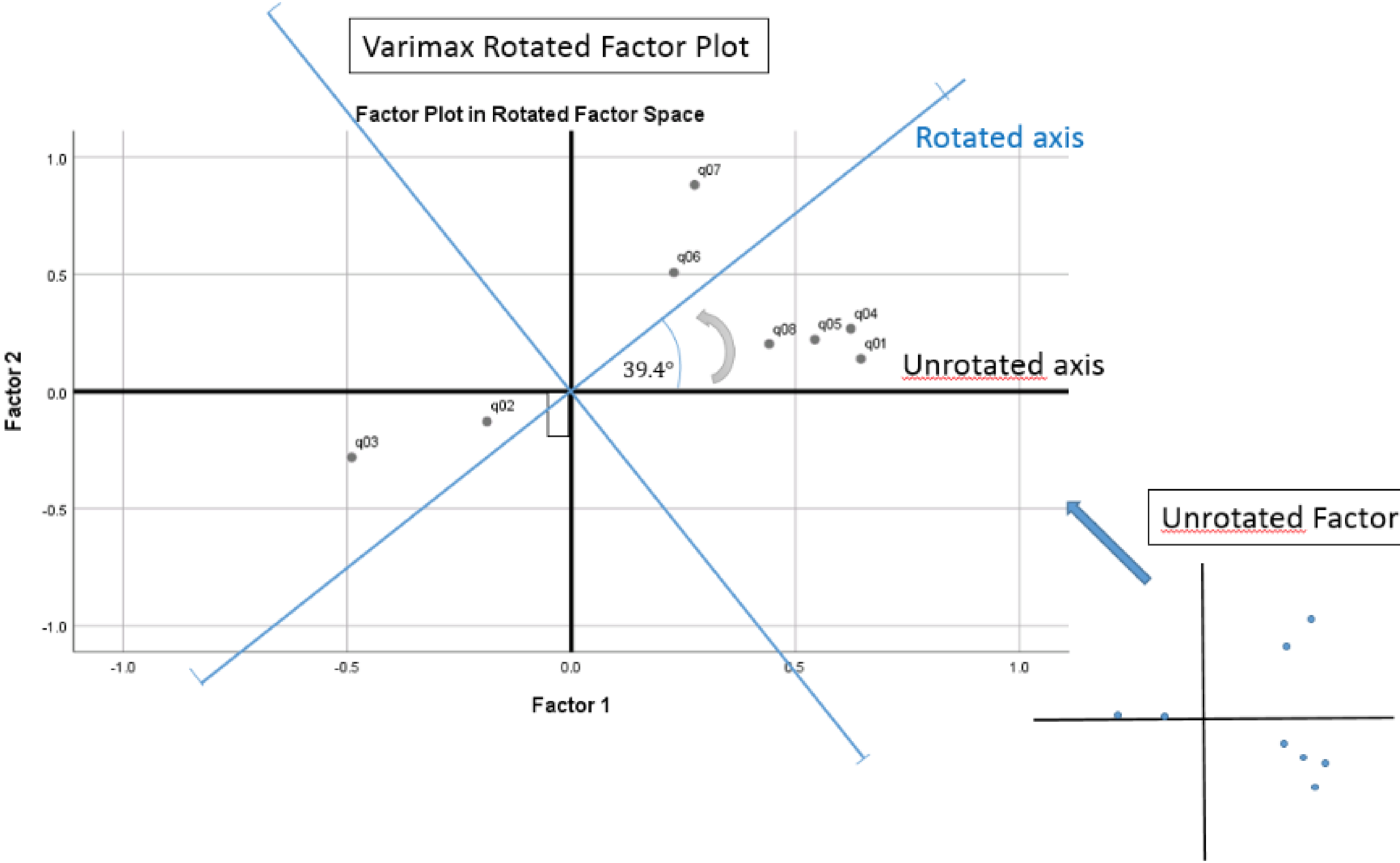
Extraction Method: Principal Axis Factoring. Rotation Method: Varimax with Kaiser Normalization.

The amount of rotation is the angle of rotation

Factor Loading Plot

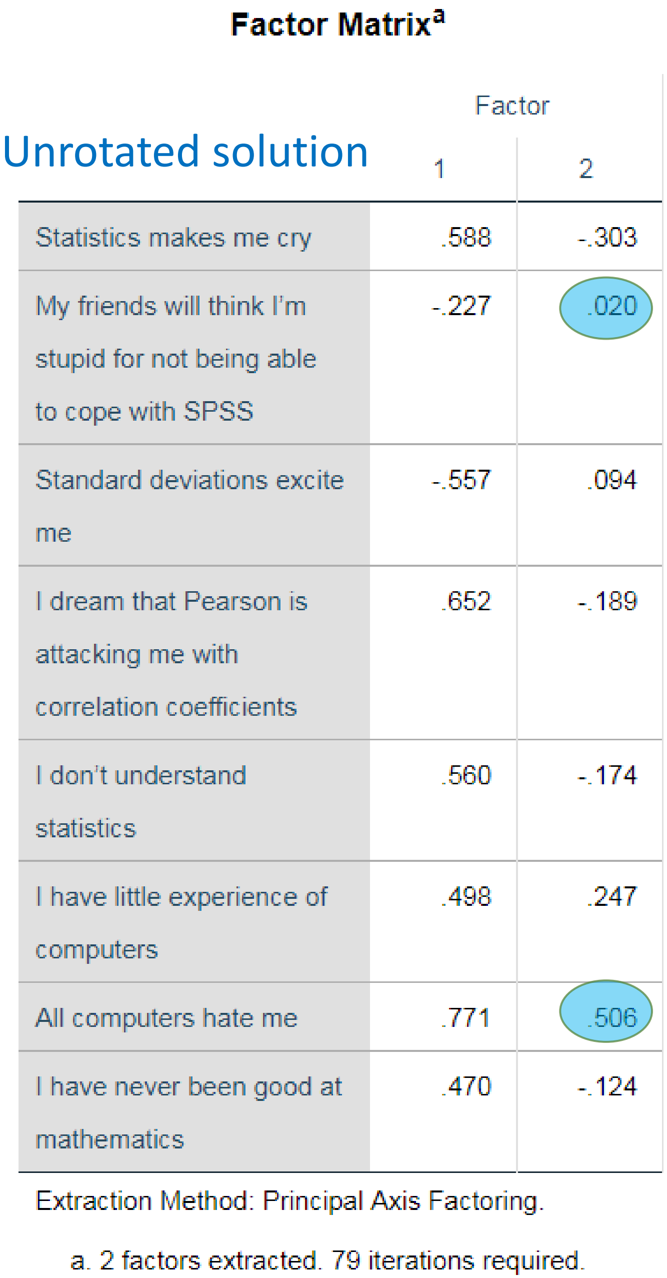
Varimax Rotated Factor Plot

Unrotated Factor Plot



Rotated Factor Matrix (2-factor PAF Varimax)

Unrotated solution



Factor Matrix[a]

	Factor	
	1	2
Statistics makes me cry	.588	-.303
My friends will think I'm stupid for not being able to cope with SPSS	-.227	.020
Standard deviations excite me	-.557	.094
I dream that Pearson is attacking me with correlation coefficients	.652	-.189
I don't understand statistics	.560	-.174
I have little experience of computers	.498	.247
All computers hate me	.771	.506
I have never been good at mathematics	.470	-.124

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 79 iterations required.

Communalities

0.438

0.052

0.319

0.461

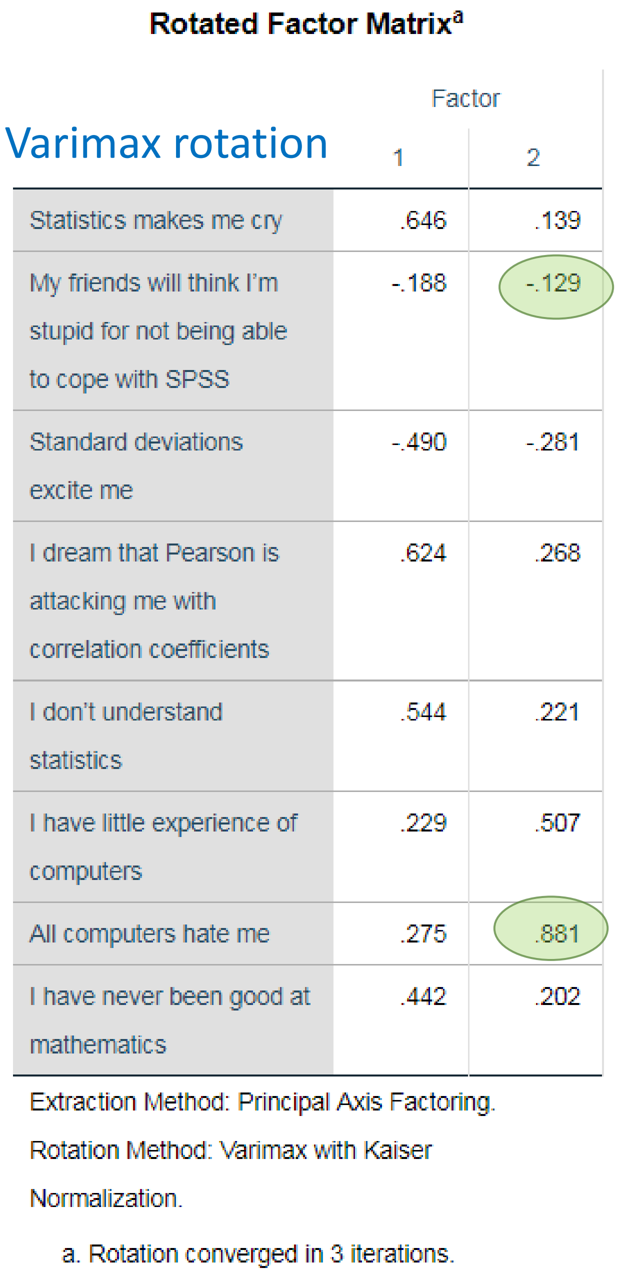
0.344

0.309

0.850

0.236

Varimax rotation



Rotated Factor Matrix[a]

	Factor	
	1	2
Statistics makes me cry	.646	.139
My friends will think I'm stupid for not being able to cope with SPSS	-.188	-.129
Standard deviations excite me	-.490	-.281
I dream that Pearson is attacking me with correlation coefficients	.624	.268
I don't understand statistics	.544	.221
I have little experience of computers	.229	.507
All computers hate me	.275	.881
I have never been good at mathematics	.442	.202

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

Communalities

0.437

0.052

0.319

0.461

0.344

0.309

0.850

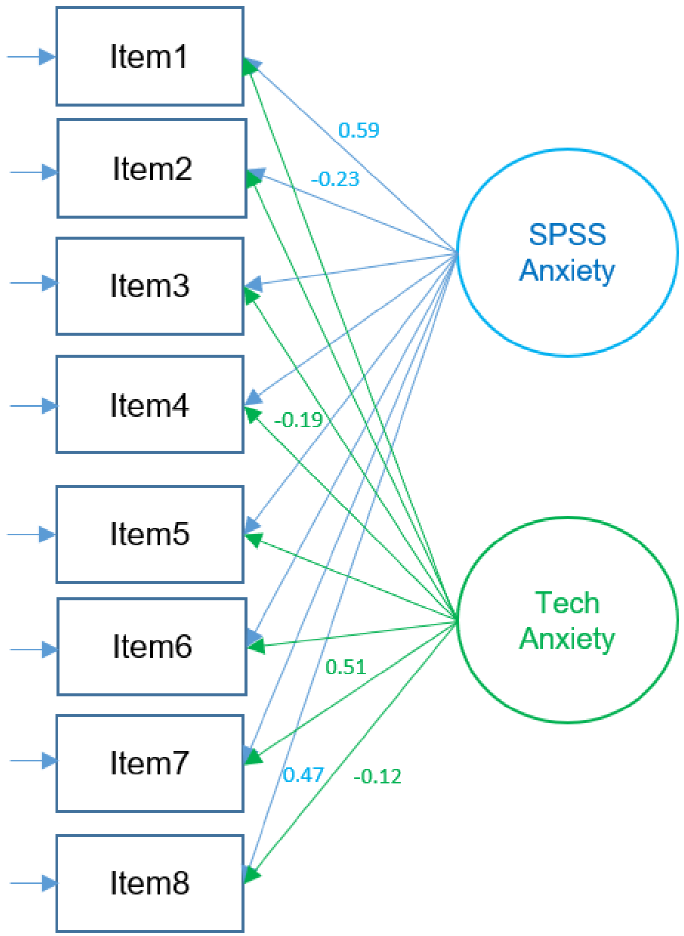
0.236

maximizes sum of the variance of squared loadings within each factor

communalities are the same

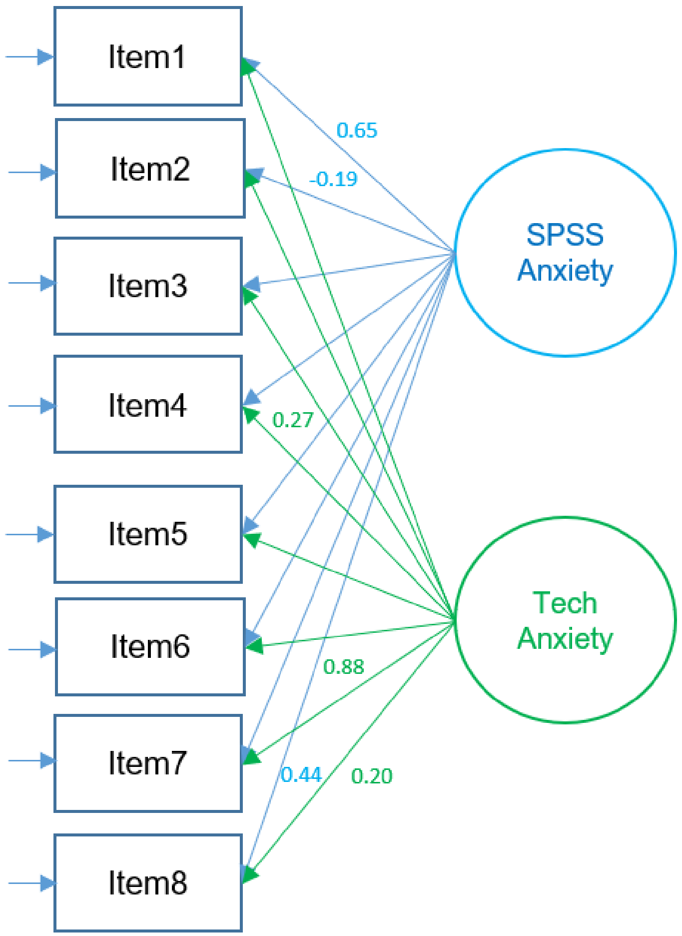
Comparing path diagrams

Unrotated Solution



Note: only selected loadings shown

Varimax Solution

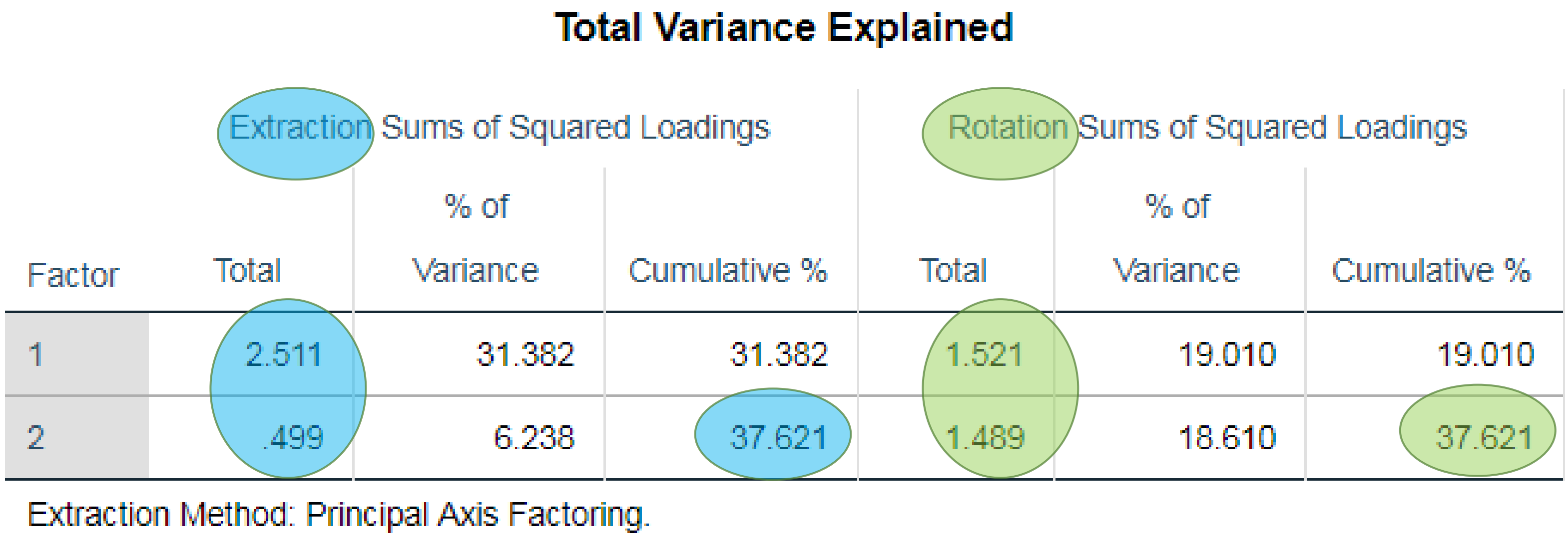


Varimax Rotation with Kaiser Normalization
Note: only selected loadings shown

Higher absolute loadings in Varimax solution for Tech Anxiety

Total Variance Explained (2-factor PAF Varimax)

True or False: Rotation changes how the variances are distributed but not the total communality



Total Variance Explained

	Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.511	31.382	31.382	1.521	19.010	19.010
2	.499	6.238	37.621	1.489	18.610	37.621

Extraction Method: Principal Axis Factoring.

maximizes variances of the loadings

3.01

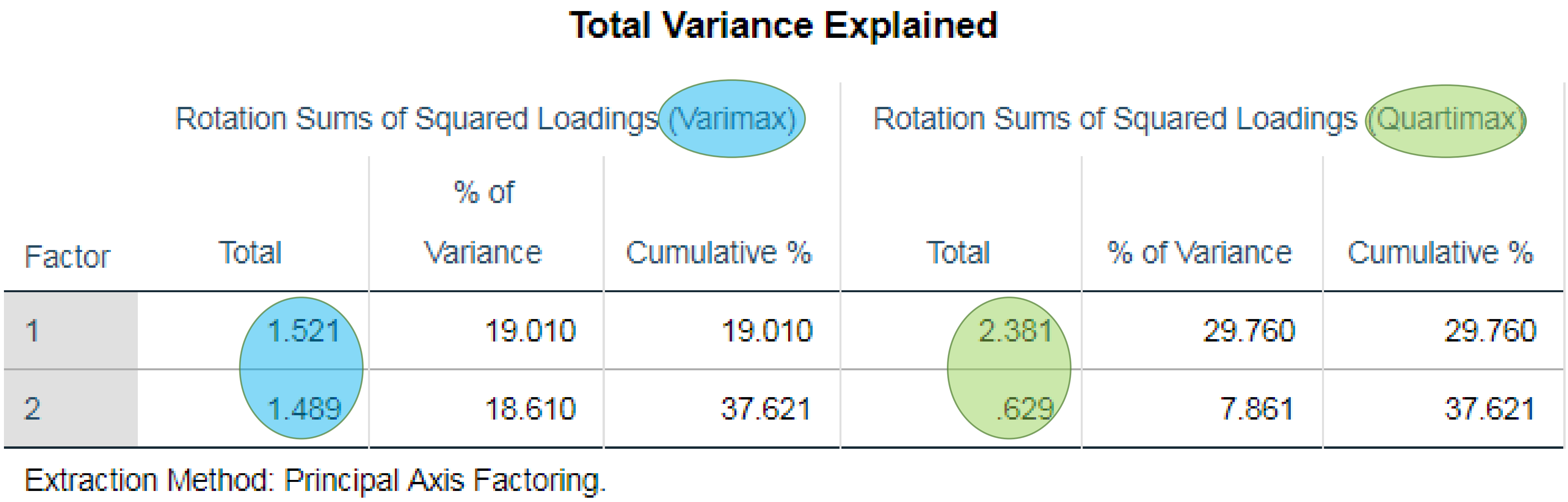
3.01

Even though the distribution of the variance is different the total sum of squared loadings is the same

Answer: T

Varimax vs. Quartimax

Quartimax: maximizes the squared loadings so that each item loads most strongly onto a single factor. Good for generating a single factor.



Total Variance Explained

	Rotation Sums of Squared Loadings (Varimax)			Rotation Sums of Squared Loadings (Quartimax)		
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.521	19.010	19.010	2.381	29.760	29.760
2	1.489	18.610	37.621	.629	7.861	37.621

Extraction Method: Principal Axis Factoring.

Varimax: good for distributing among more than one factor

The difference between Quartimax and unrotated solution is that maximum variance can be in a factor that is not the first

Oblique Rotation

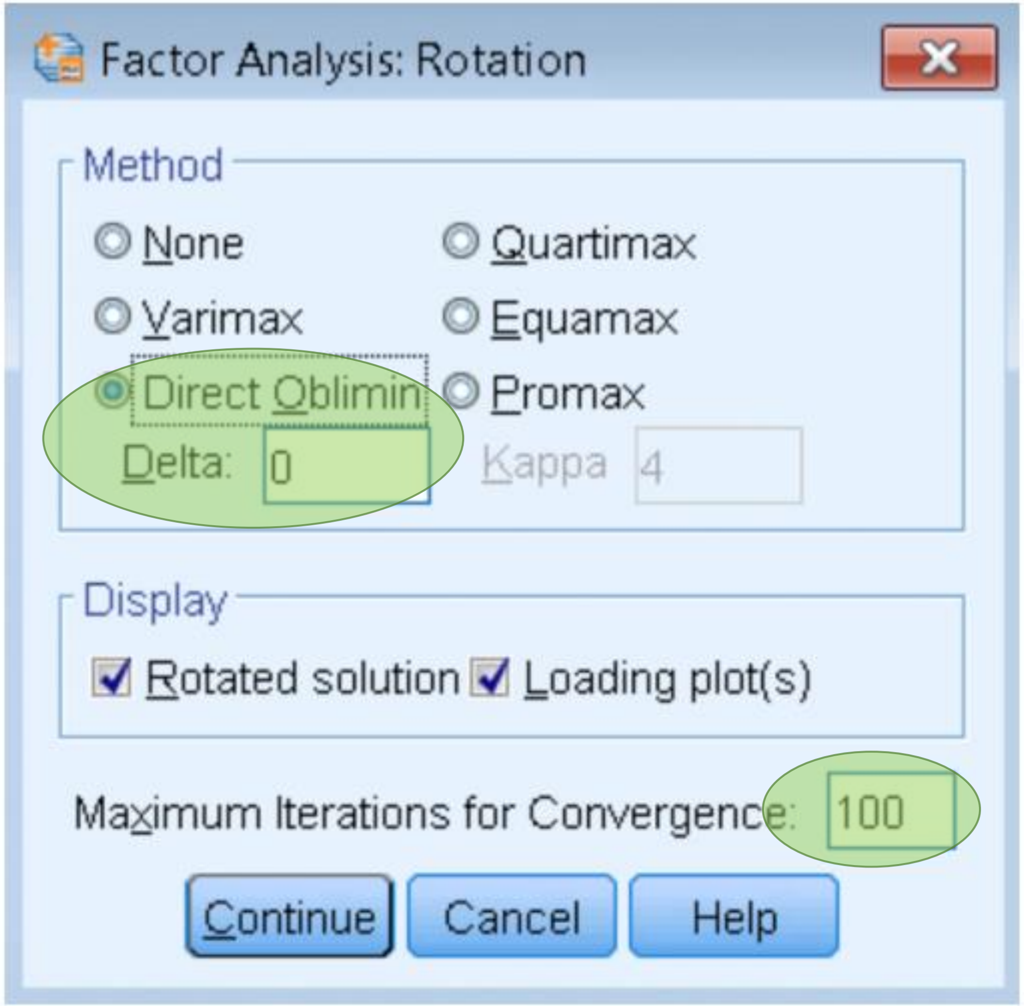
- **factor pattern matrix**
  - partial standardized regression coefficients of each item with a particular factor
    - Think (P)artial = Pattern
- **factor structure matrix**
  - simple zero order correlations of each item with a particular factor
    - Think (S)imple = Structure
- **factor correlation matrix**
  - matrix of intercorrelations among factors

Running a two-factor solution (PAF) with Quartimin

When Delta =0 → Direct Quartimin

Larger delta increases correlation among factors

Negative delta increases makes factors more orthogonal



Factor Analysis: Rotation

Method

○ None ○ Quartimax

○ Varimax ○ Equamax

● Direct Oblimin ○ Promax

Delta: 0 Kappa 4

Display

☑ Rotated solution ☑ Loading plot(s)

Maximum Iterations for Convergence: 100

Oblique rotation means the factors are correlated

Quick Check 7

1. When selecting Direct Oblimin, delta = 0 is actually Direct Quartimin. (Single Choice)

   Answer 1: T

   Answer 2: F

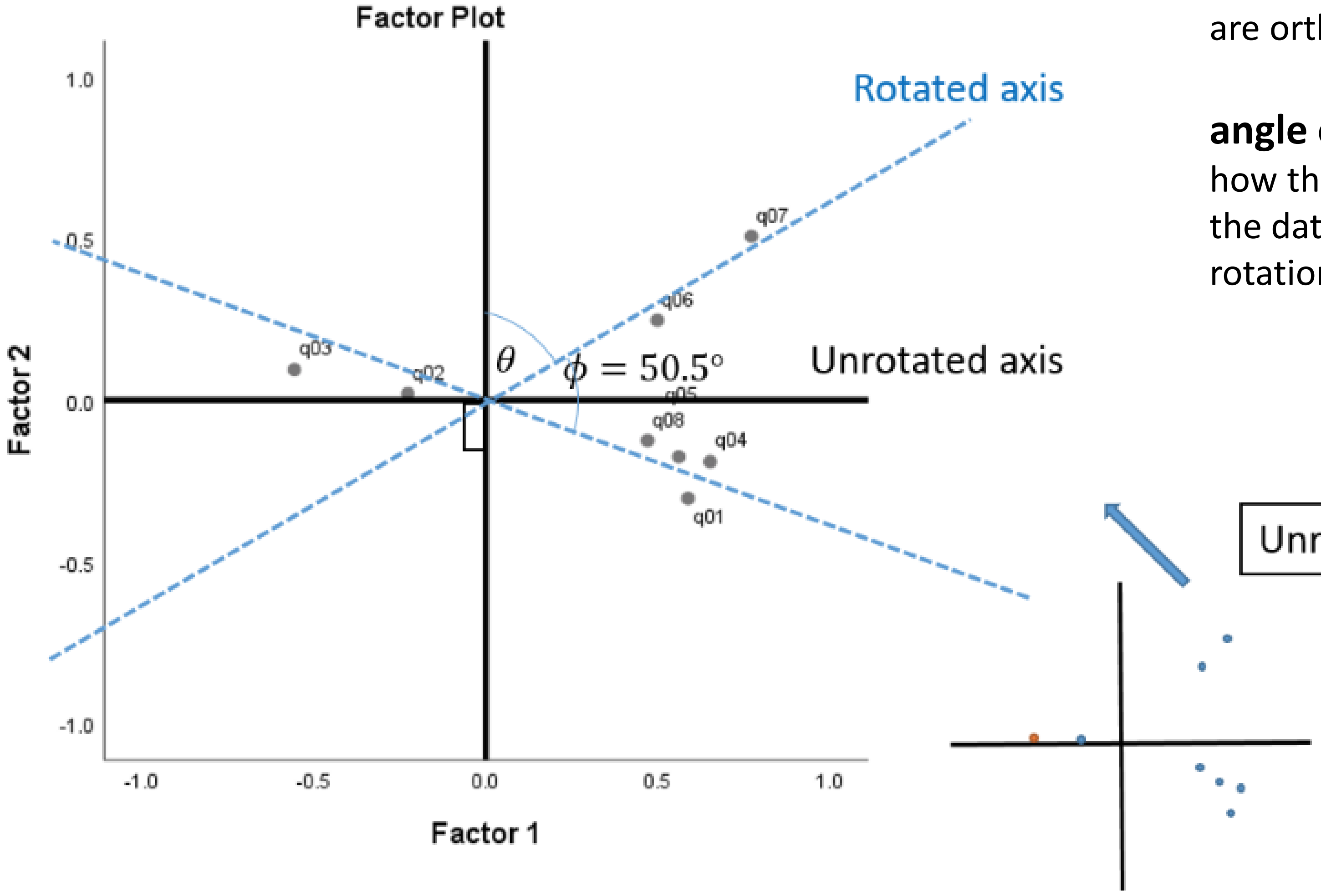
2. Smaller delta values will increase the correlations among factors. (Single Choice)

   Answer 1: T

   Answer 2: F

Factor plot of Direct Quartimin Rotation

Direct Quartimin Rotated Factor Plot



Unrotated Factor Plot

angle of correlation φ
determines whether the factors are orthogonal or oblique

angle of axis rotation θ
how the axis rotates in relation to the data points (analogous to rotation in orthogonal rotation)

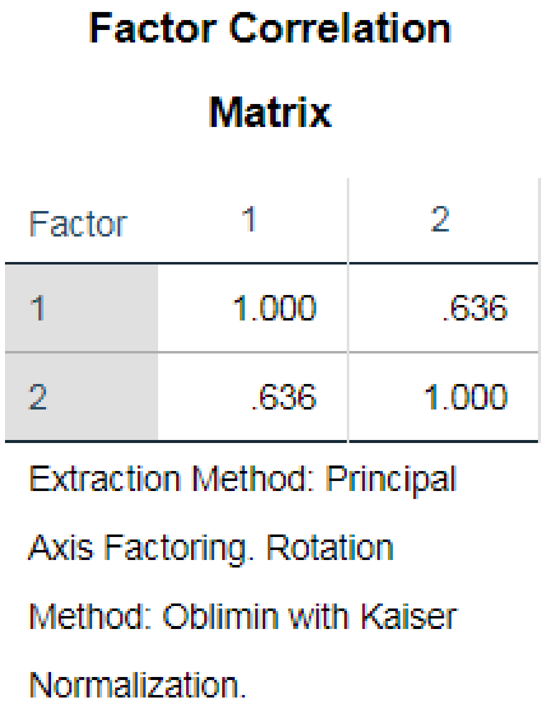
Factor Correlation Matrix (2-factor PAF Quartimin)

Factor Pattern Matrix

Factor Correlation Matrix

Factor Structure Matrix

The more correlated the factors, the greater the difference between pattern and structure matrix



Factor Correlation Matrix

Factor	1	2
1	1.000	.636
2	.636	1.000

Extraction Method: Principal Axis Factoring. Rotation Method: Oblimin with Kaiser Normalization.

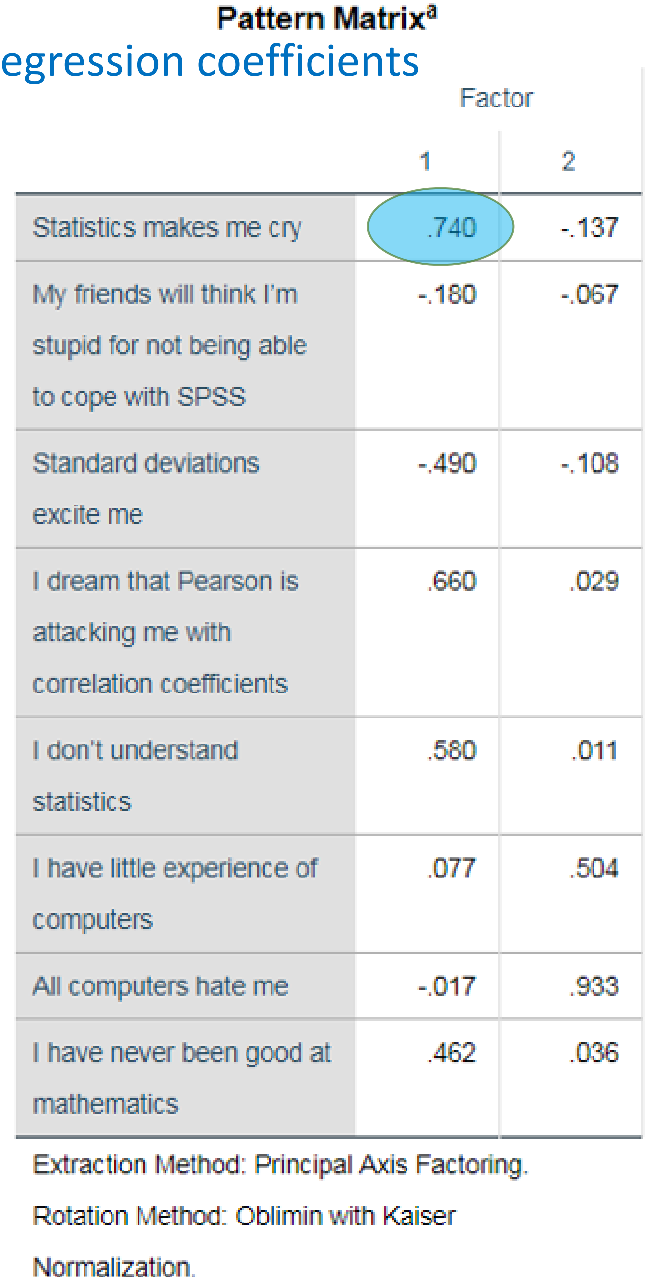
If the factors are orthogonal, the correlations between them would be zero, then the factor pattern matrix would EQUAL the factor structure matrix.

Structure & Pattern Matrix (2-factor PAF Direct Quartimin)

Partial standardized regression coefficients (can exceed one)

0.740 is the effect of Factor 1 on Item 1 *controlling* for Factor 2

There IS a way to make the sum of squared loadings equal to the communality. Think back to Orthogonal Rotation.



Pattern Matrix[a]

	Factor	
	1	2
Statistics makes me cry	.740	-.137
My friends will think I'm stupid for not being able to cope with SPSS	-.180	-.067
Standard deviations excite me	-.490	-.108
I dream that Pearson is attacking me with correlation coefficients	.660	.029
I don't understand statistics	.580	.011
I have little experience of computers	.077	.504
All computers hate me	-.017	.933
I have never been good at mathematics	.462	.036

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

0.566
0.037
0.252
0.436
0.337
0.260
0.871
0.215

Structure Matrix

	Factor	
	1	2
Statistics makes me cry	.653	.333
My friends will think I'm stupid for not being able to cope with SPSS	-.222	-.181
Standard deviations excite me	-.559	-.420
I dream that Pearson is attacking me with correlation coefficients	.678	.449
I don't understand statistics	.587	.380
I have little experience of computers	.398	.553
All computers hate me	.577	.923
I have never been good at mathematics	.485	.330

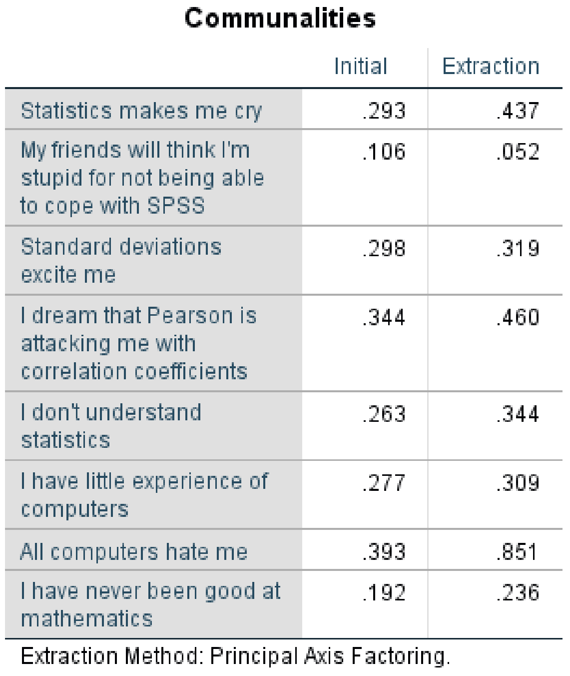
Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

0.537
0.082
0.489
0.661
0.489
0.464
1.185
0.344

Simple zero order correlations (can't exceed one)

0.653 is the simple correlation of Factor 1 on Item 1

Note that the sum of squared loadings do NOT match communalities

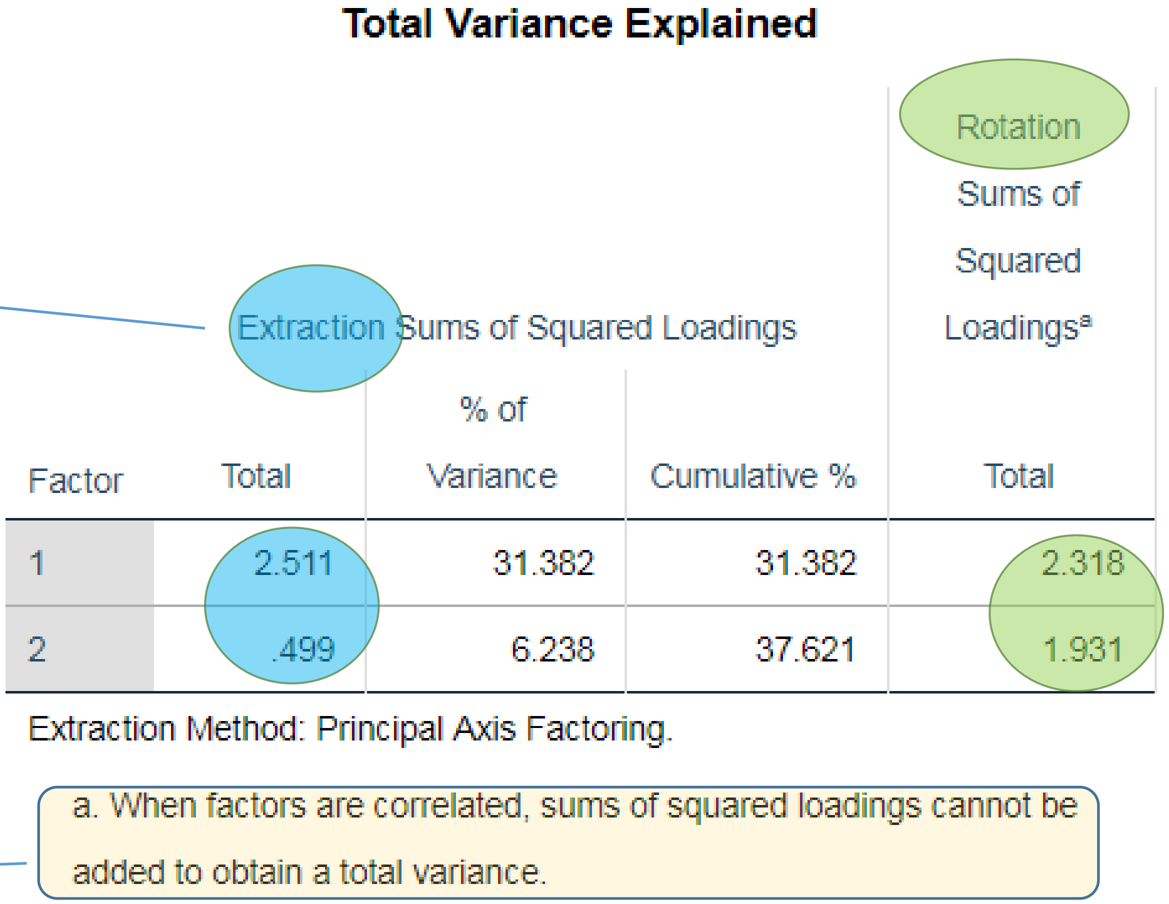


Communalities

	Initial	Extraction
Statistics makes me cry	.293	.437
My friends will think I'm stupid for not being able to cope with SPSS	.106	.052
Standard deviations excite me	.298	.319
I dream that Pearson is attacking me with correlation coefficients	.344	.460
I don't understand statistics	.263	.344
I have little experience of computers	.277	.309
All computers hate me	.393	.851
I have never been good at mathematics	.192	.236

Extraction Method: Principal Axis Factoring.

Total Variance Explained (2-factor PAF Quartimin)



Total Variance Explained

Factor	Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings^a
	Total	% of Variance	Cumulative %	Total
1	2.511	31.382	31.382	2.318
2	.499	6.238	37.621	1.931

Extraction Method: Principal Axis Factoring.

a. When factors are correlated, sums of squared loadings cannot be added to obtain a total variance.

This is exactly the same as the unrotated 2-factor PAF solution

SPSS uses the structure matrix to calculate this
-factor contributions will overlap and become greater than the total variance

Note: now the sum of the squared loadings is HIGHER than the unrotated solution

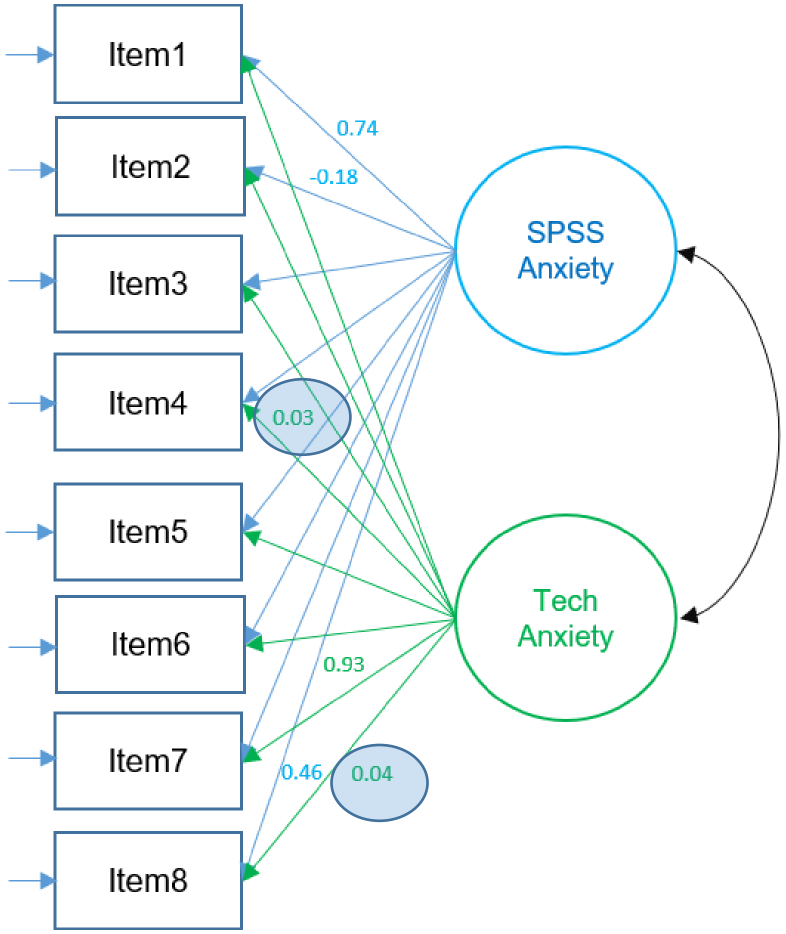
SPSS uses the structure matrix to calculate this
-factor contributions will overlap and become greater than the total variance

3.01

4.25

Comparing Pattern to Structure Matrix

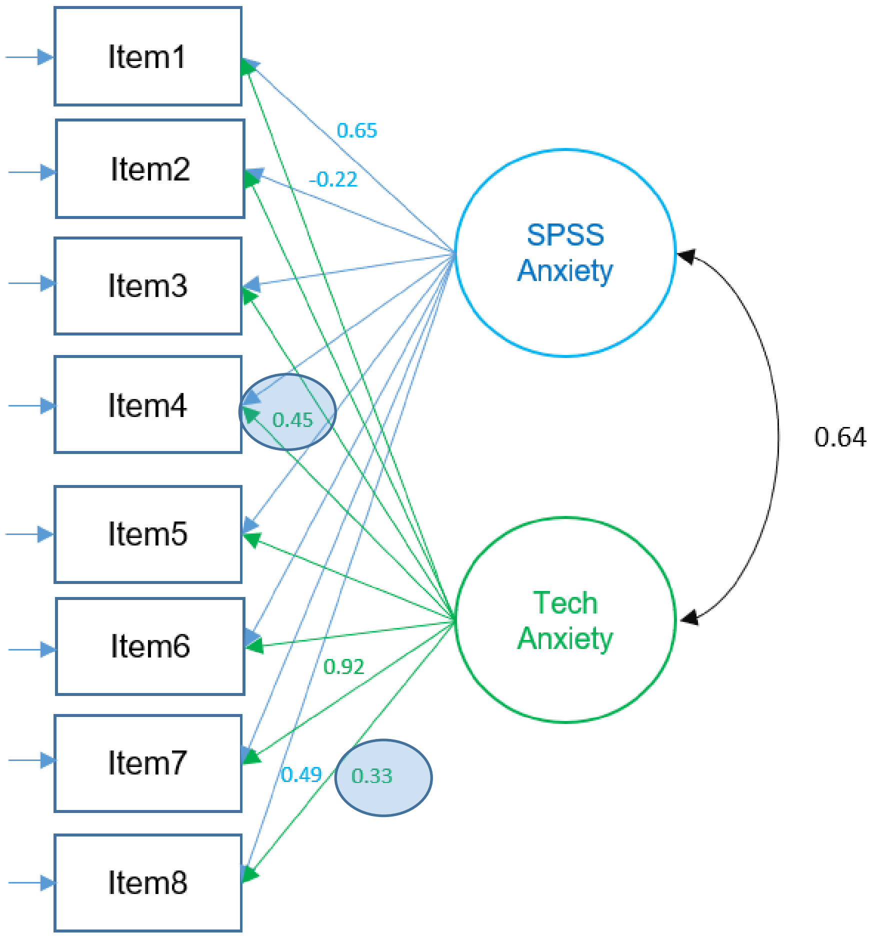
Partial loadings



Direct Quartimin Rotation with Kaiser Normalization
Pattern Matrix (only selected loadings shown)

Correlations same

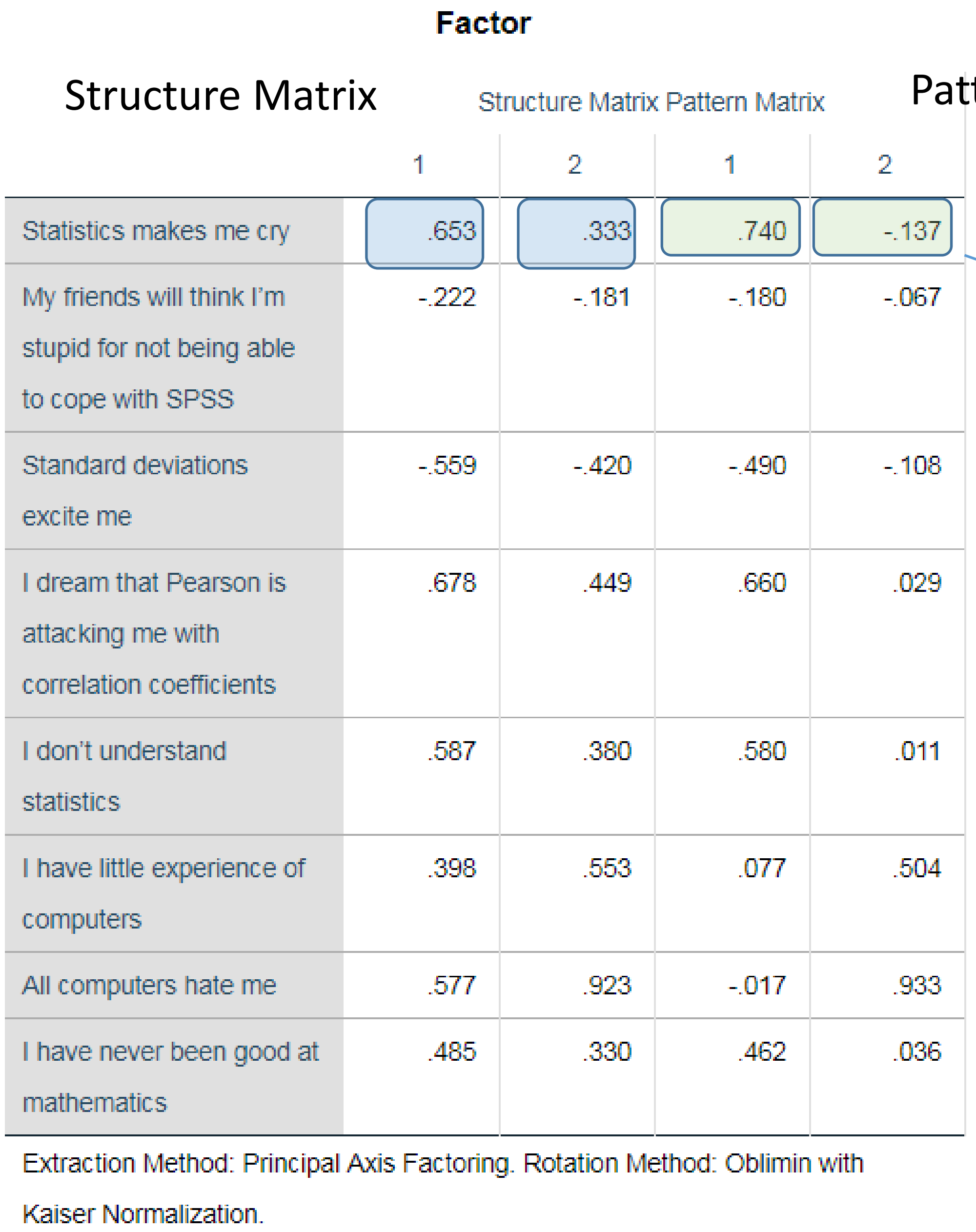
Zero-order loadings



Direct Quartimin Rotation with Kaiser Normalization
Structure Matrix (only selected loadings shown)

Lower absolute loadings of Items 4,8 onto Tech Anxiety for Pattern Matrix

Interpreting loadings (2-factor PAF Quartimin)



Structure Matrix

Pattern Matrix

	Factor			
	Structure Matrix		Pattern Matrix	
	1	2	1	2
Statistics makes me cry	.653	.333	.740	-.137
My friends will think I'm stupid for not being able to cope with SPSS	-.222	-.181	-.180	-.067
Standard deviations excite me	-.559	-.420	-.490	-.108
I dream that Pearson is attacking me with correlation coefficients	.678	.449	.660	.029
I don't understand statistics	.587	.380	.580	.011
I have little experience of computers	.398	.553	.077	.504
All computers hate me	.577	.923	-.017	.933
I have never been good at mathematics	.485	.330	.462	.036

Extraction Method: Principal Axis Factoring. Rotation Method: Oblimin with Kaiser Normalization.

Why do you think the second loading is lower in the Pattern Matrix compared to the Structure Matrix?

Factor or Pattern Matrix?

- There is no consensus about which one to use in the literature

- Hair et al. (1995)
  - Better to interpret the **pattern** matrix because it gives the unique contribution of the factor on a particular item
- Pett et al. (2003)
  - Structure matrix should be used for interpretation
  - Pattern matrix for obtaining factor scores

- My belief: I agree with Hair

Hair, J. F. J., Anderson, R. E., Tatham, R. L., & Black, W. C. (1995). Multivariate data analysis . Saddle River.

Pett, M. A., Lackey, N. R., & Sullivan, J. J. (2003). *Making sense of factor analysis: The use of factor analysis for instrument development in health care research*. Sage.

Quick Check 8

1. In oblique rotation, an element of a factor pattern matrix is the unique contribution of the factor to the item whereas an element in the factor structure matrix is the non-unique contribution to the factor to an item. (Single Choice)

Answer 1: T

Answer 2: F

2. In the Total Variance Explained table, the Rotation Sum of Squared Loadings represent the unique contribution of each factor to total common variance. (Single Choice)

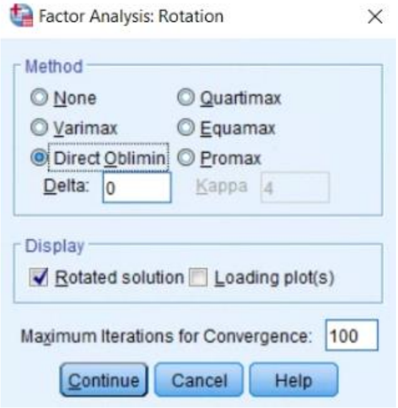
Answer 1: T

Answer 2: F

3. If the factors are orthogonal, then the Pattern Matrix equals the Structure Matrix (Single Choice)

Answer 1: T

Answer 2: F



Factor Analysis: Rotation

Method

○ None ○ Quartimax
○ Varimax ○ Equamax
● Direct Oblimin ○ Promax
Delta: 0 Kappa 4

Display

☑ Rotated solution ☐ Loading plot(s)

Maximum Iterations for Convergence: 100

4. Varimax, Quartimax and Equamax are three types of orthogonal rotation and Direct Oblimin, Direct Quartimin and Promax are three types of oblique rotations. (Single Choice)

Answer 1: T

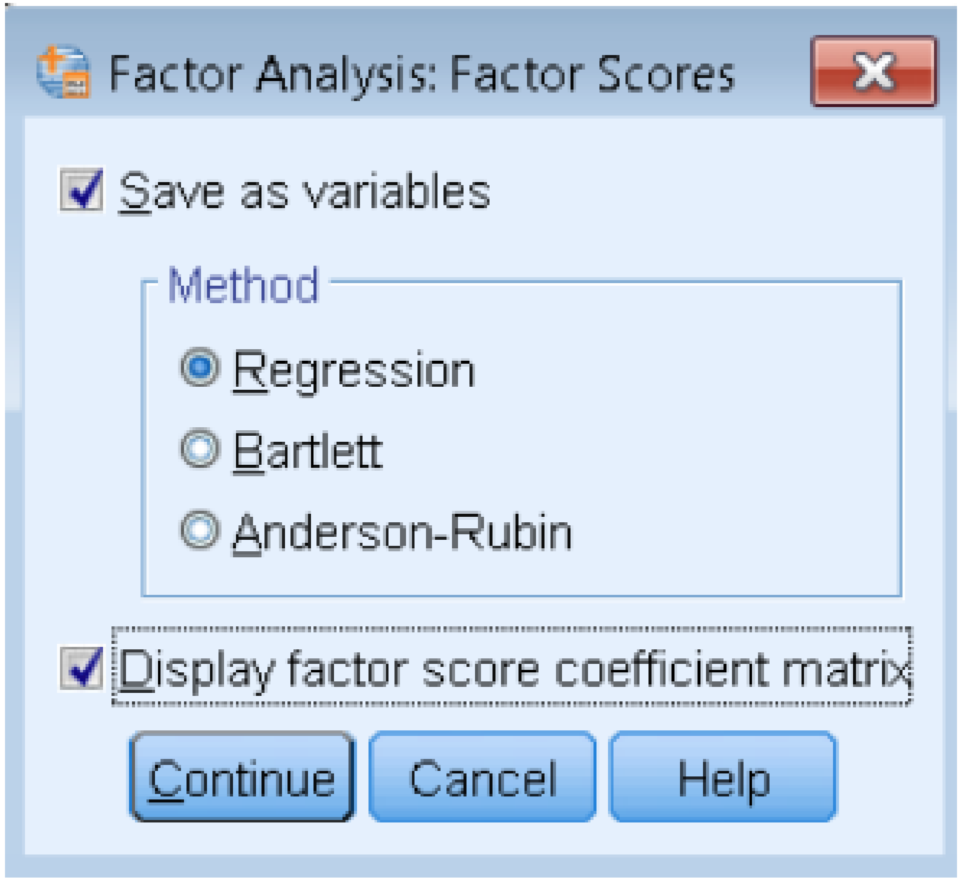
Answer 2: F

Generating Factor Scores

- Regression
- Bartlett
- Anderson-Rubin

Generating factor scores (Quartimin, Reg Method)

Analyze – Dimension Reduction – Factor – Factor Scores



Factor Analysis: Factor Scores

☑ Save as variables

Method

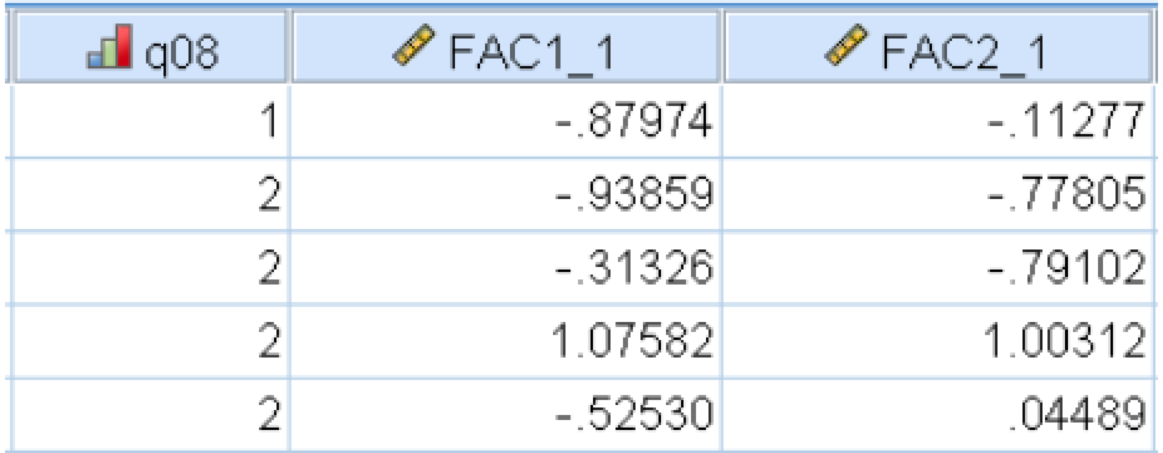
● Regression

○ Bartlett

○ Anderson-Rubin

☑ Display factor score coefficient matrix

What it looks like in SPSS Data View



q08	FAC1_1	FAC2_1
1	-.87974	-.11277
2	-.93859	-.77805
2	-.31326	-.79102
2	1.07582	1.00312
2	-.52530	.04489

Factor Score Coefficient Matrix (Direct Quartimin)

This is how the factor scores are generated

SPSS takes the standardized scores for each item
Then multiply each score

Factor Score Coefficient Matrix

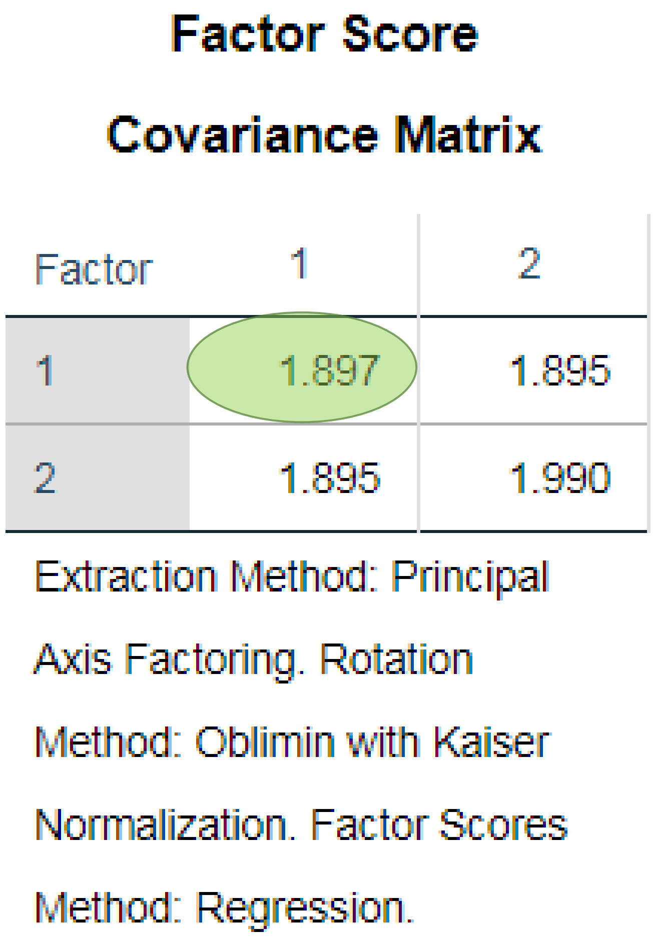
	Factor	
	1	2
Statistics makes me cry	.284	.005
My friends will think I'm stupid for not being able to cope with SPSS	-.048	-.019
Standard deviations excite me	-.171	-.045
I dream that Pearson is attacking me with correlation coefficients	.274	.045
I don't understand statistics	.197	.036
I have little experience of computers	.048	.095
All computers hate me	.174	.814
I have never been good at mathematics	.133	.028

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization. Factor Scores Method: Regression.

-0.452	-0.452
-0.733	-0.733
1.32	1.32
-0.829	-0.829
-0.749	-0.749
-0.203	-0.203
0.0692	0.0692
-1.42	-1.42
-0.880	**-0.113**

Factor Score Covariance (Direct Quartimin)

Covariance matrix of the **true** factor scores

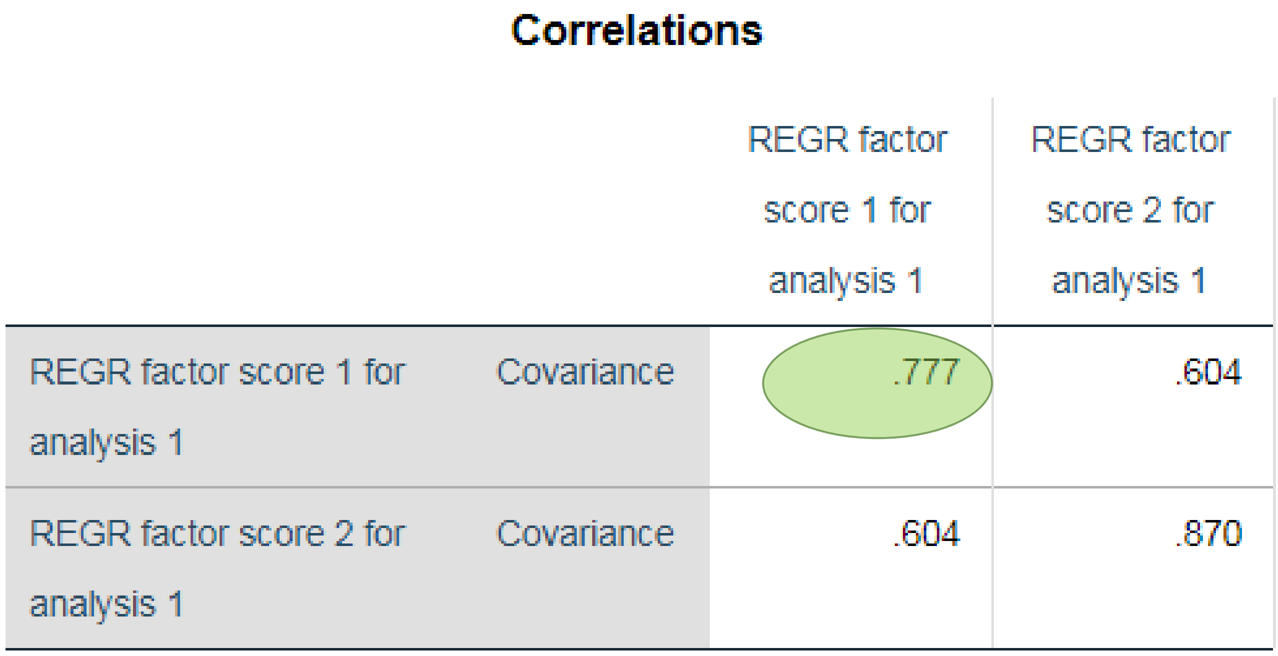


Factor Score Covariance Matrix

Factor	1	2
1	1.897	1.895
2	1.895	1.990

Extraction Method: Principal Axis Factoring. Rotation Method: Oblimin with Kaiser Normalization. Factor Scores Method: Regression.

Covariance matrix of the **estimated** factor scores



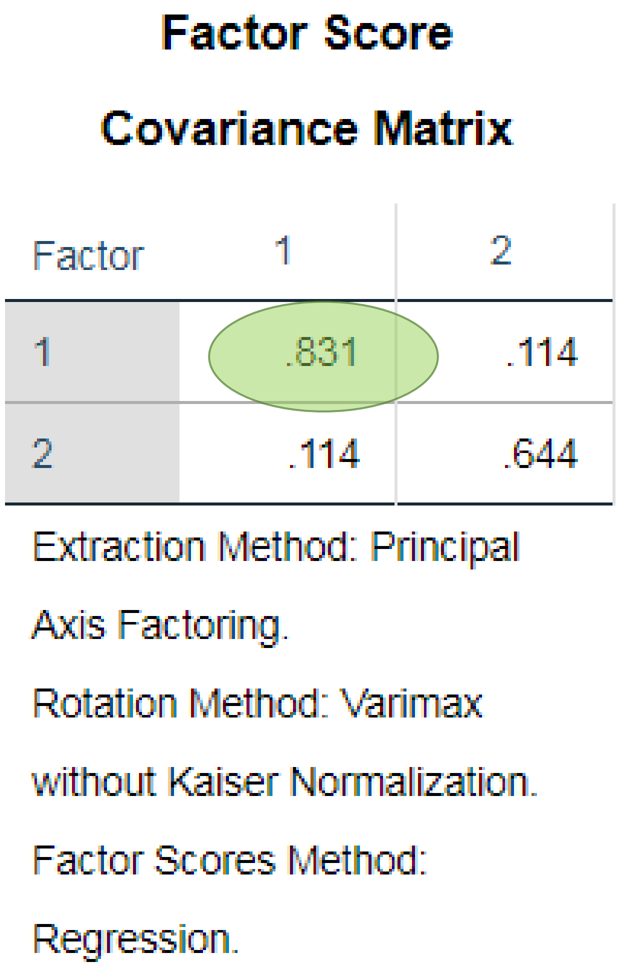
Correlations

		REGR factor score 1 for analysis 1	REGR factor score 2 for analysis 1
REGR factor score 1 for analysis 1	Covariance	.777	.604
REGR factor score 2 for analysis 1	Covariance	.604	.870

Notice that for Direct Quartimin, the raw covariances do not match

Regression method has factor score mean of zero, and variance equal to the squared multiple correlation of estimated and true factor scores

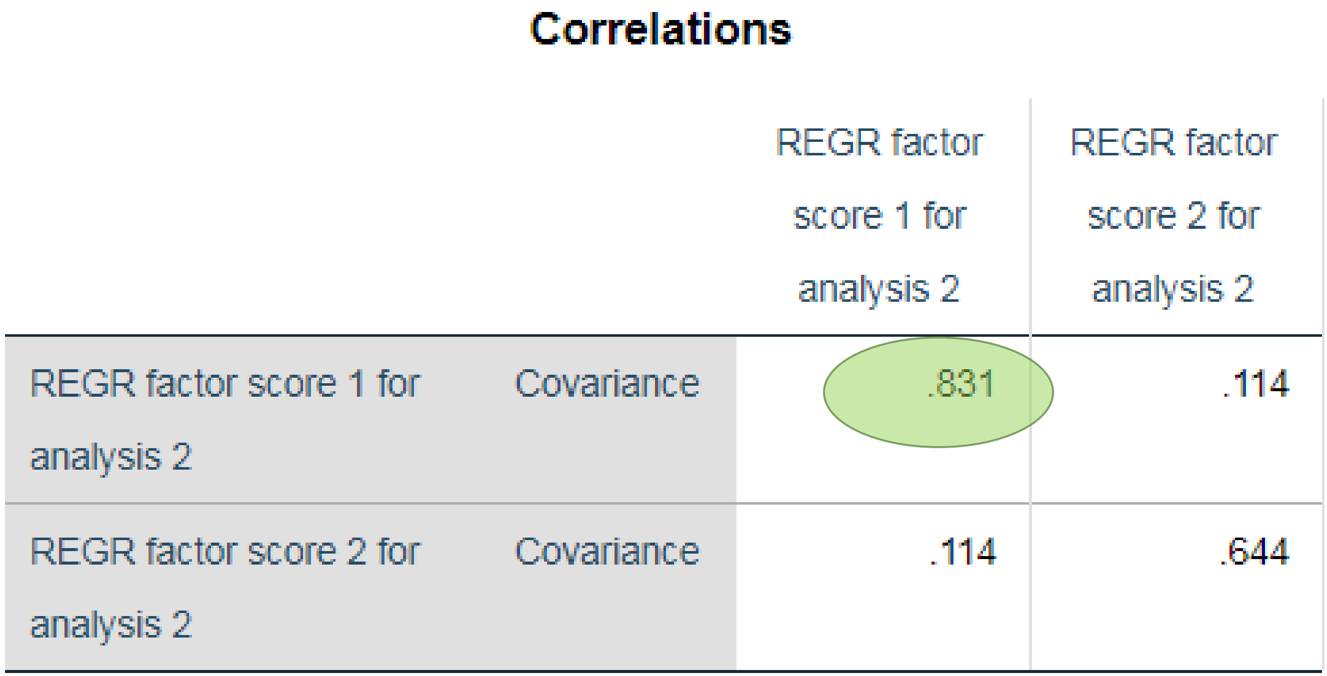
Factor Score Covariance (Varimax)



Factor Score Covariance Matrix

Factor	1	2
1	.831	.114
2	.114	.644

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax without Kaiser Normalization.
Factor Scores Method: Regression.



Correlations

		REGR factor score 1 for analysis 2	REGR factor score 2 for analysis 2
REGR factor score 1 for analysis 2	Covariance	.831	.114
REGR factor score 2 for analysis 2	Covariance	.114	.644

Notice that for Direct Quartimin, the raw correlations *do* match (property of Regression method)

However, note that the factor scores are still correlated even though we did Varimax

Comparing score methods

- 1. Regression Method
  - Variance equals the square multiple correlation between factors and variables
  - Maximizes correlation between estimated and true factor scores but can be **biased**
- 2. Bartlett
  - Factor scores highly correlate with own true factor and not with others
  - **Unbiased** estimate of true factor scores
- 3. Anderson-Rubin
  - Estimated factor scores become uncorrelated with other true factors and uncorrelated with other estimated factor scores
  - **Biased** especially if factors are actually correlated, not for oblique rotations

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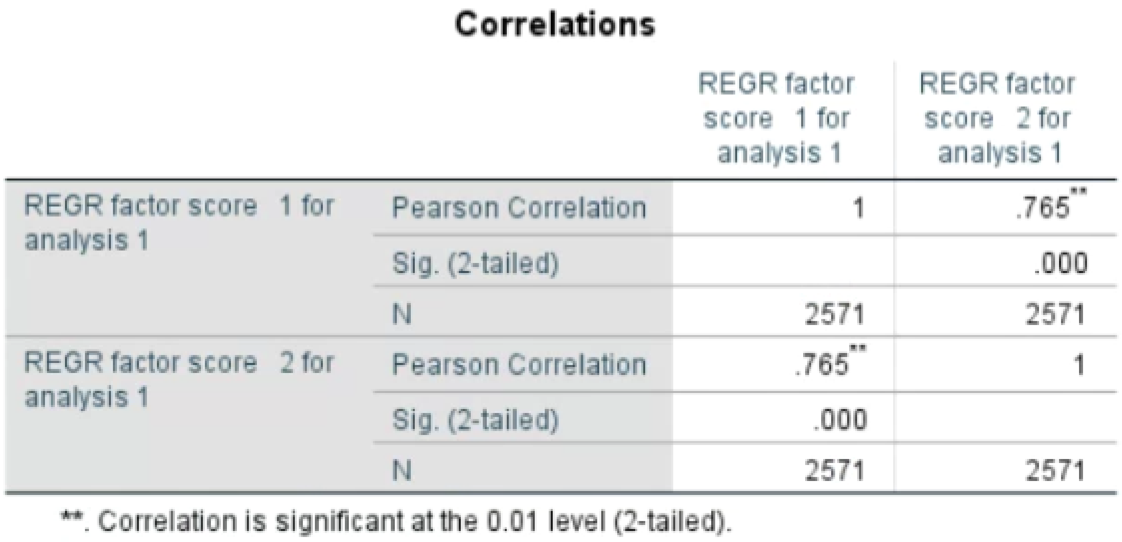
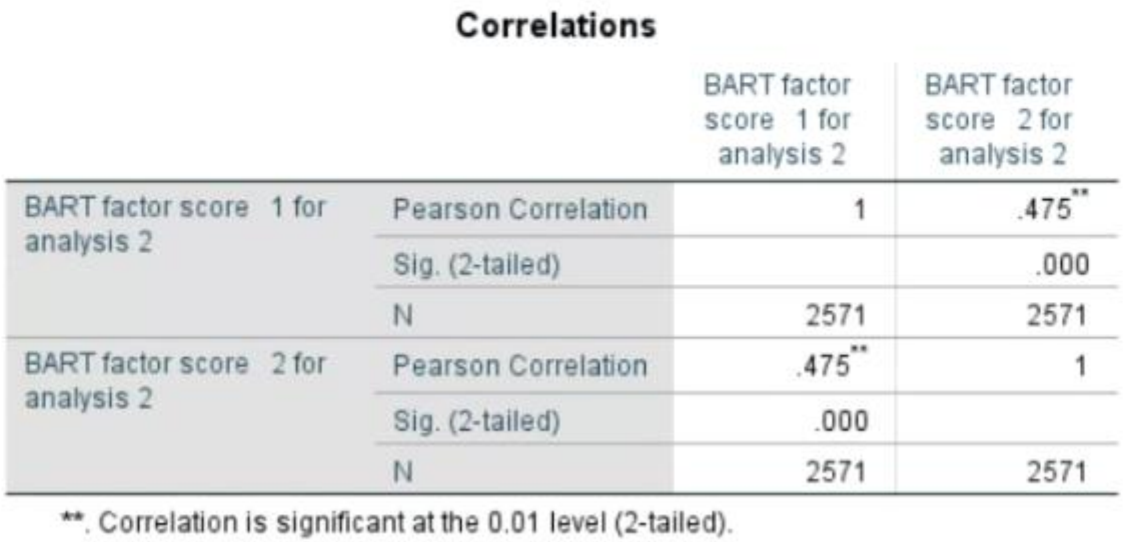
Correlations between estimated factors



Correlations

		REGR factor score 1 for analysis 1	REGR factor score 2 for analysis 1
REGR factor score 1 for analysis 1	Pearson Correlation	1	.765**
	Sig. (2-tailed)		.000
	N	2571	2571
REGR factor score 2 for analysis 1	Pearson Correlation	.765**	1
	Sig. (2-tailed)	.000	
	N	2571	2571

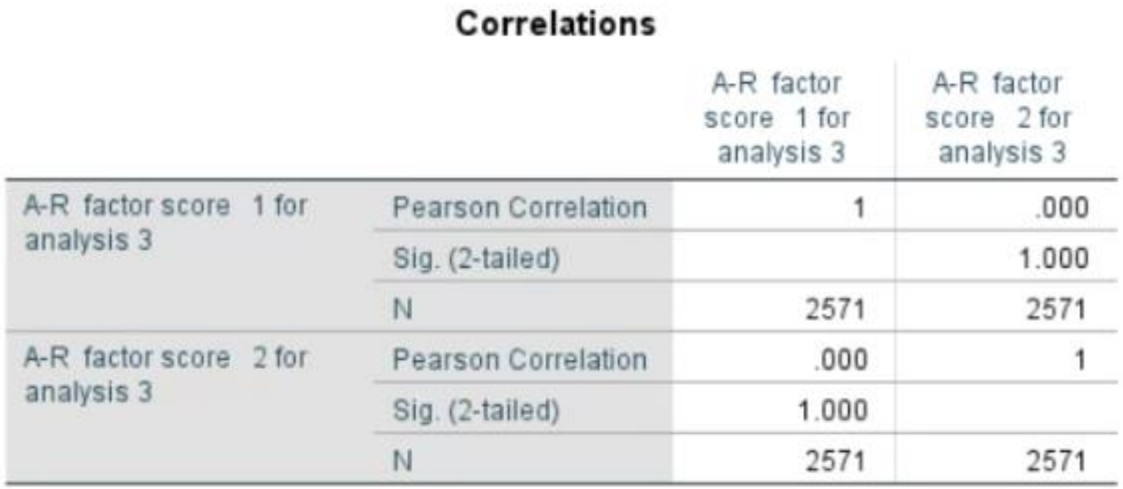
**. Correlation is significant at the 0.01 level (2-tailed).



Correlations

		BART factor score 1 for analysis 2	BART factor score 2 for analysis 2
BART factor score 1 for analysis 2	Pearson Correlation	1	.475**
	Sig. (2-tailed)		.000
	N	2571	2571
BART factor score 2 for analysis 2	Pearson Correlation	.475**	1
	Sig. (2-tailed)	.000	
	N	2571	2571

**. Correlation is significant at the 0.01 level (2-tailed).



Correlations

		A-R factor score 1 for analysis 3	A-R factor score 2 for analysis 3
A-R factor score 1 for analysis 3	Pearson Correlation	1	.000
	Sig. (2-tailed)		1.000
	N	2571	2571
A-R factor score 2 for analysis 3	Pearson Correlation	.000	1
	Sig. (2-tailed)	1.000	
	N	2571	2571

Direct Quartimin

Quick Check 9

1. If you want the highest correlation of the factor score with the corresponding factor (i.e., highest validity), choose the regression method. (Single Choice)

Answer 1: T

Answer 2: F

2. Bartlett scores are unbiased whereas Regression and Anderson-Rubin scores are biased. (Single Choice)

Answer 1: T

Answer 2: F

3. Anderson-Rubin is appropriate for orthogonal but not for oblique rotation because factor scores will be uncorrelated with other factor scores. (Single Choice)

Answer 1: T

Answer 2: F